**Eksempel 7 i § 10.6**:

Parameterkurven er gitt ved: $x(t) := t$ $y(t) := t^2$ $z(t) := 0$

Vektorform av parameterkurven: $r(t) := \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$

Hastighetsvektoren:

$$v(t) := \begin{pmatrix} \frac{d}{dt}x(t) \\ \frac{d}{dt}y(t) \\ \frac{d}{dt}z(t) \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 2\cdot t \\ 0 \end{pmatrix}$$

$v(t)_0 \rightarrow 1$

Farten:

$$fart(t) := \sqrt{\left(v(t)_0\right)^2 + \left(v(t)_1\right)^2 + \left(v(t)_2\right)^2} \rightarrow \left(1 + 4\cdot t^2\right)^{\frac{1}{2}}$$

Enhets-hastighetsvektor: $T(t) := \frac{v(t)}{fart(t)}$ simplify $\rightarrow \begin{bmatrix} \frac{1}{\left(1 + 4\cdot t^2\right)^{\frac{1}{2}}} \\ 2\cdot\frac{t}{\left(1 + 4\cdot t^2\right)^{\frac{1}{2}}} \\ 0 \end{bmatrix}$

T(t) derivert:

$$dT(t) := \begin{pmatrix} \frac{d}{dt}T(t)_0 \\ \frac{d}{dt}T(t)_1 \\ \frac{d}{dt}T(t)_2 \end{pmatrix} \text{ simplify } \rightarrow \begin{bmatrix} \frac{-4}{\left(1 + 4\cdot t^2\right)^{\frac{3}{2}}}\cdot t \\ \frac{2}{\left(1 + 4\cdot t^2\right)^{\frac{3}{2}}} \\ 0 \end{bmatrix}$$

Lengde av T(t) derivert:

$$dTlengde(t) := \sqrt{\left(dT(t)_0\right)^2 + \left(dT(t)_1\right)^2 + \left(dT(t)_2\right)^2} \rightarrow 2\cdot\left[\frac{4}{\left(1 + 4\cdot t^2\right)^3}\cdot t^2 + \frac{1}{\left(1 + 4\cdot t^2\right)^3}\right]^{\frac{1}{2}}$$

Krumning:

$$\kappa(t) := \frac{dTlengde(t)}{fart(t)} \rightarrow 2\cdot\frac{\left[\frac{4}{\left(1+4\cdot t^2\right)^3}\cdot t^2+\frac{1}{\left(1+4\cdot t^2\right)^3}\right]^{\frac{1}{2}}}{\left(1+4\cdot t^2\right)^{\frac{1}{2}}}$$

Krumningsradius:

$$\rho(t) := \frac{1}{\kappa(t)} \rightarrow \frac{1}{2\cdot\left[\frac{4}{\left(1+4\cdot t^2\right)^3}\cdot t^2+\frac{1}{\left(1+4\cdot t^2\right)^3}\right]^{\frac{1}{2}}}\cdot\left(1+4\cdot t^2\right)^{\frac{1}{2}}$$

Prinsipal enhetsnormal:

$$N(t) := \frac{dT(t)}{dTlengde(t)} \rightarrow \begin{bmatrix} \frac{-2}{\left(1+4\cdot t^2\right)^{\frac{3}{2}}}\cdot\frac{t}{\left[\frac{4}{\left(1+4\cdot t^2\right)^3}\cdot t^2+\frac{1}{\left(1+4\cdot t^2\right)^3}\right]^{\frac{1}{2}}} \\ \frac{1}{\left(1+4\cdot t^2\right)^{\frac{3}{2}}\cdot\left[\frac{4}{\left(1+4\cdot t^2\right)^3}\cdot t^2+\frac{1}{\left(1+4\cdot t^2\right)^3}\right]^{\frac{1}{2}}} \\ 0 \end{bmatrix}$$

Sentrum i den "osculating" sirkelen er gitt ved (a(t),b(t)) der (se boka side 845):

$$a(t) := r(t)_0 + \rho(t)\cdot N(t)_0 \rightarrow t - \frac{1}{\left[\frac{4}{\left(1+4\cdot t^2\right)^3}\cdot t^2+\frac{1}{\left(1+4\cdot t^2\right)^3}\right]\cdot\left(1+4\cdot t^2\right)}\cdot t$$

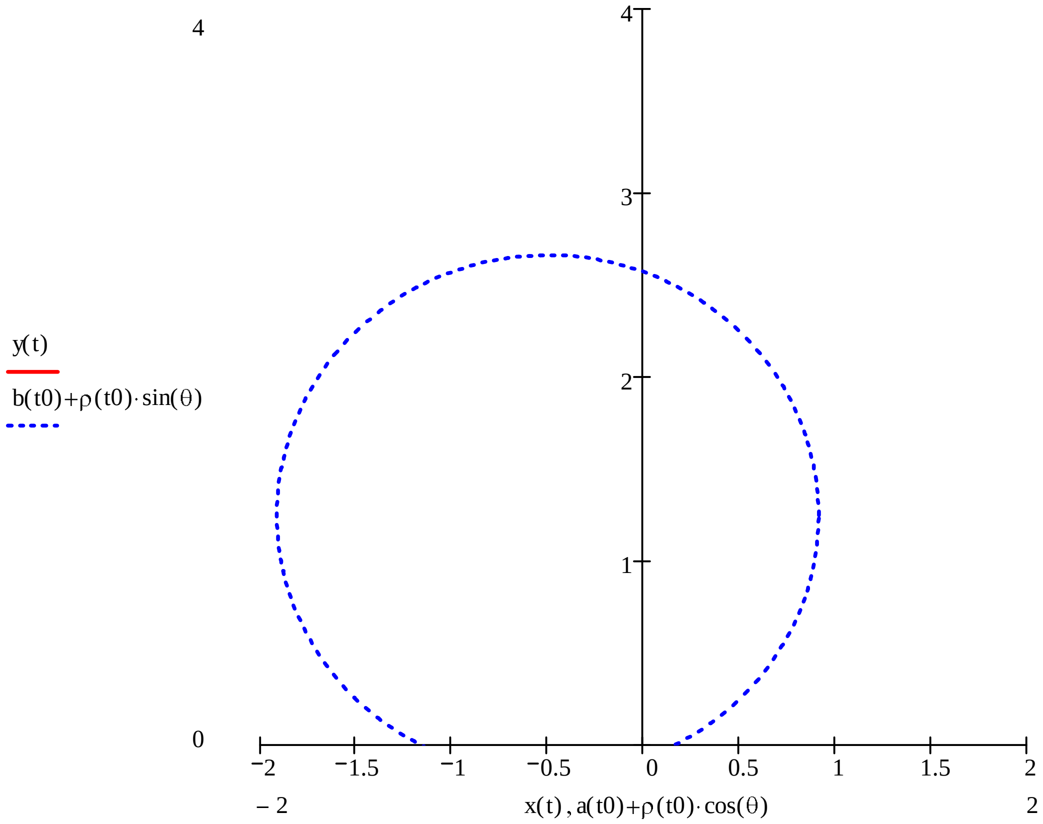
$$b(t) := r(t)_1 + \rho(t)\cdot N(t)_1 \rightarrow t^2 + \frac{1}{2\cdot\left[\frac{4}{\left(1+4\cdot t^2\right)^3}\cdot t^2+\frac{1}{\left(1+4\cdot t^2\right)^3}\right]\cdot\left(1+4\cdot t^2\right)}$$

$\theta := 0, 0.04 .. 2\cdot\pi$ $t0 := 0.5$

$\rho(t0) = 1.414$ $a(t0) = -0.5$ $b(t0) = 1.25$

$x(t0) = 0.5$ $y(t0) = 0.25$

Den "osculating" sirkelen har radius 1.414 og sentrum i (-0.5 , 1.25) når t=0.5.
Tangeringspunktet er i (0.5 , 0.25).



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