

ALTA: Solving Equations in Grade 5

Background

This ALTA involves three important transitions:

- From solving arithmetic equations to solving algebraic equations
- From the equal sign marking the answer of a calculation to the equal sign representing the equality of two expressions
- From concrete representations of unknown to abstract representations

Arithmetic and algebraic equations

Arithmetic equations can be solved by operating on the numbers in the equation. For example, $3x + 2 = 5$ is solved by subtracting 2 and dividing by 3. Such equations are not difficult for children to solve. Algebraic equations must be solved by operating on the unknown in the equation. For example, $3x + 2 = 5x$ is solved by subtracting $3x$ and dividing by 2. Such equations are difficult for children to solve. We have identified eight levels of equation complexity.

	1	2	3	4
Arithmetic (R)	$X = B$	$X + A = B$	$AX = B$	$AX + B = C$
Algebraic (L)	$AX = BX + C$ ($A - B = 1$)	$AX = BX + C$ ($A - B > 1$)	$AX + B = CX + D$ ($A - C = 1$)	$AX + B = CX + D$ ($A - C > 1$)

The equal sign

In primary school children use the equal sign mostly to mark the answer of a calculation. $2 + 2 = 4$ can be read as “two plus two *makes* four”. There is a direction to the sign. The answer comes after it. “Four makes two plus two” doesn’t make sense. An essential part of algebraic thinking is thinking about the relationships between expressions. The simplest relationship is equality. Equality is bidirectional. $2 + 2 = 4$ means $2 + 2$ is the same quantity as 4, and it can be read either way. 4 is also the same quantity as $2 + 2$.

From concrete to abstract

Many people associate algebraic thinking with the use of letters to stand for numbers. It is possible to think algebraically without using letters. Until the 1500s mathematicians used words and pictures to represent their algebraic thinking. This ALTA is structured around a gradual transition from concrete objects to abstract symbols.

In the ALTA the representations used are:

- A balance (a physical balance, not a drawing or computer simulation of one)
- Algebra tiles (rectangles representing numbers and variables)
- Drawings of algebra tiles
- Symbols (x , y , etc.)



A central assumption is that the transition from one representation to another should be as gradual as possible. At first algebra tiles are introduced as a way of representing what is happening on the balance. Later they become a representation that can be operated on directly. Drawings at first are used to record what is done with algebra tiles. Later they become a representation that can be operated on directly. Symbols at first are used to record what is done with algebra tiles or drawings. Later they become a representation that can be operated on directly.

Curriculum connections

Core elements

Reasoning and argumentation

Reasoning in mathematics means the ability to follow, assess and understand mathematical chains of thought. It means that the pupils shall understand that mathematical rules and results are not random, but have clear and logical grounds. The pupils shall formulate their own reasoning to understand and to solve problems. Argumentation in mathematics means that the pupils give grounds for their methods, reasoning and solutions, and prove that these are valid.

Representation and communication

Representation in mathematics means ways of expressing mathematical concepts, relationships and problems. Representations can be concrete, contextual, visual, verbal and symbolic. Communication in mathematics refers to the pupils using mathematical language in conversations, argumentation and reasoning. The pupils shall have the opportunity to use mathematical representations in different contexts through their own experiences and in mathematical conversations. The pupils shall have the opportunity to explain and give grounds for the choice of representation form. The pupils must be able to switch between mathematical representations and everyday language and to switch between different representations.

Abstraction and generalisation

Abstraction in mathematics means gradually developing formalisation of thoughts, strategies and mathematical language. The development goes from concrete descriptions to formal symbol language and formal reasoning. Generalisation in mathematics refers to the pupils finding relationships and structures, without being presented a finished solution. This means that the pupils can explore numbers, calculations and figures to find relationships, and then formalise by using algebra and well-reasoned representations.

Grade 5 competence goals

- solve equations and inequalities through logical reasoning and explain what it means that a number is a solution of an equation

Structure of the ALTA

	Representation: Equations	Balance	Tiles	Drawings (recording)	Symbols (recording)	Drawings (operated on)	Symbols (operated on)
R1	$X = B$	Session 1	Session 2 Group work	Session 2 Plenum	Session 2 teacher		
R2	$X + A = B$						
R3	$AX = B$						
R4	$AX + B = C$						
L1	$AX = BX + C$ ($A - B = 1$)						
L2	$AX = BX + C$ ($A - B > 1$)						
L3	$AX + B = CX + D$ ($A - C = 1$)						
L4	$AX + B = CX + D$ ($A - C > 1$)						

	Representation: Equations	Balance	Tiles	Drawings (recording)	Symbols (recording)	Drawings (operated on)	Symbols (operated on)
R1	$X = B$						
R2	$X + A = B$						
R3	$AX = B$						
R4	$AX + B = C$		Session 3				Session 3 possible
L1	$AX = BX + C$ ($A - B = 1$)						
L2	$AX = BX + C$ ($A - B > 1$)						
L3	$AX + B = CX + D$ ($A - C = 1$)						
L4	$AX + B = CX + D$ ($A - C > 1$)						

	Representation: Equations	Balance	Tiles	Drawings (recording)	Symbols (recording)	Drawings (operated on)	Symbols (operated on)
R1	$X = B$						
R2	$X + A = B$						
R3	$AX = B$						
R4	$AX + B = C$		Session 4 if needed				Session 4
L1	$AX = BX + C$ ($A - B = 1$)						
L2	$AX = BX + C$ ($A - B > 1$)						
L3	$AX + B = CX + D$ ($A - C = 1$)						
L4	$AX + B = CX + D$ ($A - C > 1$)						

First Session: Physical experiences with balances

Materials:

- A balance for each group.
- Many identical objects as units for weighing.
- Several objects of different weights, to be weighed.
- Several identical objects stuck together to be weighed as a group.

Activity:

1. Pupils take an object of unknown weight and find out how much it weighs in terms of the units (can be repeated with a new object). (Level R1)
2. Pupils take a group of objects stuck together and find out the weight of the group and each object in the group (can be repeated with a different group of objects). (Level R2)
3. Pupils place an object of unknown weight and some units on one side, and find out the weight of the object with the units and the object alone (can be repeated with a new object). (Level R3)
4. Pupils place a group of objects stuck together and some units on one side, and find out the weight of the group with the units, the group alone and each object in the group (can be repeated with a different group of objects). (Level R4)
5. Pupils are asked to formulate rules for keeping the balance equal, when removing or adding units.

Discussion:

What strategies did the students use to maintain their balance?

How have they found the weight when a group of objects is stuck together? (the teacher can represent this as $3x = 48 \rightarrow x = 16$)

How have they found the weight when extra units are with an object? (the teacher can represent this as $x + 3 = 18 \rightarrow x = 15$)

What rules have they found for keeping the balance equal? (record these: remove the same from both sides; add the same to both sides; (maybe) divide both sides.)

Second Session: Manipulation of physical objects that represent the balance (algebra tiles)

Materials:

- Algebra tiles
- Equation-mats

Activity:

The students are presented with algebra tiles and an equation-mat. The equation-mat represents the balance and the variable tile represents an object of unknown weight. The unit tiles represent the units used for weighing. Through the manipulation of tiles, students will solve equations of different levels, increasing in the degree of algebraic reasoning required from R2 to L4. Level R1 is not used in this session.

Sample Tasks:

(<https://www.didax.com/apps/algebra-tiles/> or <https://technology.cpm.org/general/tiles/> can be used to make images)

Level R2

- $1 + 2 = X$



Level R3

- $3 + X = 5$



Level R4

- $2X = 4$



Level L1

- $2X = 4 + X$



Level L2

- $3X = X + 6$



Level L3

- $3X + 2 = 2X + 7$

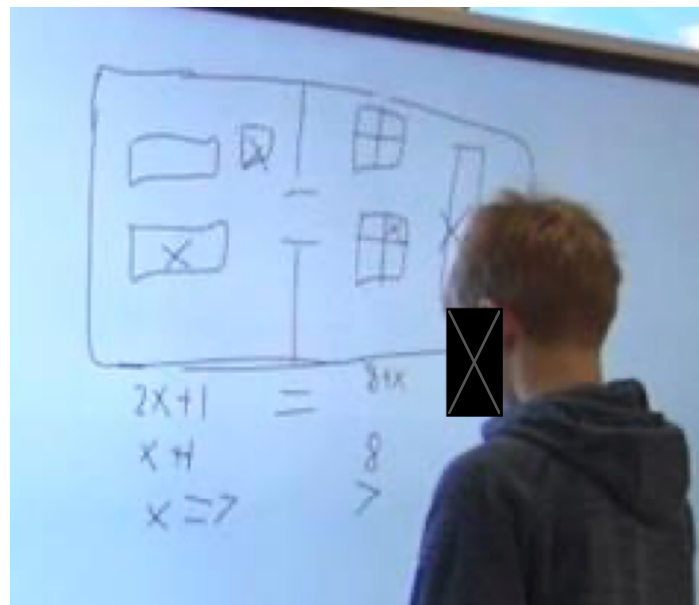


Level L4

- $3X + 3 = X + 7$



After solving these tasks individually or in groups, have pupils share their solutions in plenum, recording their actions using pictures, labeled with symbols suggested by the teacher (see Figure).



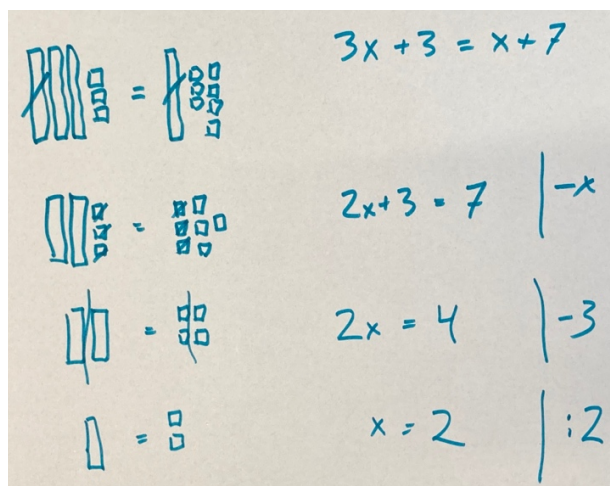
Third session: Drawings of physical objects that represent the balance

Materials:

- Algebra tiles
- Equation-mats

Activity:

The pupils should solve equations represented by symbols using tiles. They should record each step of their solutions using a 'storyboard'.



They should solve equations of types R4 and L1–4:

R4	$3x + 3 = 9$	$4x + 2 = 14$
L1	$3x = 2x + 25$	$5x = 4x + 7$
L2	$4x = 2x + 4$	$6x = 3x + 9$
L3	$3x + 3 = 2x + 5$	$5x + 8 = 4x + 10$
L4	$3x + 3 = x + 7$	$5x + 3 = 2x + 12$

After solving these tasks individually or in groups, have pupils share their solutions in plenum, working directly on pictures labeled with symbols.

Fourth Session: Symbolic solutions of equations

Materials:

- Algebra tiles (if needed)
- Equation-mats (if needed)

Activity:

During this session the aim is for the students to solve equations using symbolic notation.

Make sure that the tasks here are "difficult" enough, so that the students do not immediately see what the answer is.

Storyboards should be made, but drawings of tiles are optional.

They should solve equations of types R4 and L1–4. Larger numbers should be used for some equations, so they are difficult to do with tiles.

R4	$7x + 5 = 26$	$30x + 6 = 66$
L1	$4x = 3x + 5$	$5x = 4x + 17$
L2	$3x = x + 6$	$6x = 3x + 18$
L3	$x + 9 = 2x + 7$	$5x + 8 = 4x + 10$
L4	$4x + 2 = 2x + 10$	$5x + 3 = 2x + 12$