

ALTA Grade 5 Fractions (revision 1, 21 January 2024)

This ALTA is focussed on establishing the basic rules for operations on fractions, based on meaningful explorations, and explaining those rules using mathematical reasoning and argumentation.

This ALTA is structured around the idea of the pupils “acting like mathematicians”. This means that they engage in *exploring*, *generalising* and *reasoning and argumentation*.

Mathematics in the making resembles any other human knowledge in the making. You have to guess a mathematical theorem before you prove it; ... You have to combine observations and follow analogies. You have to try and to try again.” (Polya 1954, p. vi)

In Lesson 1 they are also engage in *defining* which involves *abstracting* from several examples, and then generalising their definition to all fractions.

The pupils will generalise and explain several general mathematical rules (Theorems). These should be recorded somewhere visible (a poster?) during the lessons.

Outline

Lesson 1: Defining Unit Fractions, nevner, comparing unit fractions

Definition: A [unit] fraction is one of several parts the same size.

Theorem: The bigger the nevner the smaller the fraction.

Lesson 2: Defining Fractions, teller, comparing fractions

Definition: A fraction is one or more of several parts the same size.

Theorem: Two fractions are equal if there is a number K such that $\text{nevner1} \cdot K = \text{nevner2}$ and $\text{teller1} \cdot K = \text{teller2}$.

Lesson 3: Folding the number line

Theorems: If the tellere are the same, then the bigger the nevner, the closer to 0.

A fraction can be written as a percent or a decimal. The percent is the teller of the equivalent fraction with nevner 100. The decimal is that teller with 0, in front.

Lesson 4: Parts of an amount

Theorem: $\frac{a}{b}$ of a set of c things is $\frac{a \cdot c}{b}$ things.

Lessons from the Grade 7 ALTA could also be adapted for Grade 5.

Kjerneelementer: Utforskning og problemløsning, Abstraksjon og generalisering, Resonnering og argumentasjon, Representasjon og kommunikasjon

Kompetansemål etter 5. trinn: Mål for opplæringen er at eleven skal kunne:

- utforske og forklare sammenhenger mellom brøker, desimaltall og prosent og bruke det i hoderegning
- beskrive brøk som del av en hel, som del av en mengde og som tall på tallinjen og vurdere og navngi størrelsene
- representere brøker på ulike måter og oversette mellom de ulike representasjonene
- utvikle og bruke ulike strategier for regning med positive tall og brøk og forklare tenkemåtene sine

Lesson 1: Defining Unit Fractions, comparing unit fractions

Introduction (whole class)

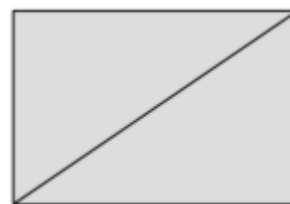
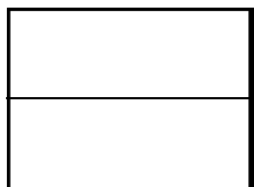
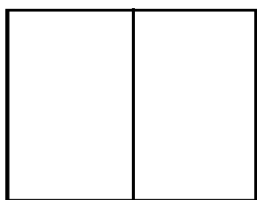
“For the next N lessons you are going to be acting like mathematicians. This means you are going to be finding patterns, you are going to be turning those patterns into general rules and you are going to be using mathematical reasoning and argumentation to explain why those rules always work.”

Exploring a half

(whole class & working with partners in pairs) [Materials: A4 paper]

Ask the kids “What is one half”, ask them to bring examples from their daily life. Work with the contexts that the kids bring. Discuss one half of different wholes. Then move on to the paper folding context.

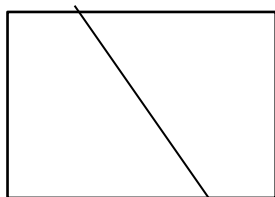
“Fold a piece of paper in half. Try to do it a different way than your neighbour is doing it. Mark the fold with a marker so we can see where you folded” Collect and display different foldings. Discuss what makes it one half. Discuss one half of the same whole.



Challenge them to fold a half differently.

Defining a half

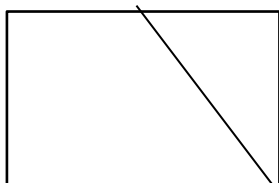
“One thing mathematicians do is the *define* things. In mathematics a definition needs to say exactly what a thing is, so we can use the definition to reason about it. How would you define a *half*?”



One of two parts, one of two identical parts, one of two parts the same size.

Write possible definitions on board.

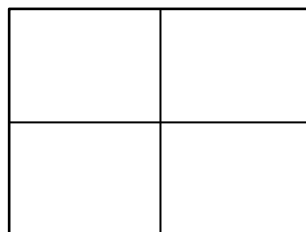
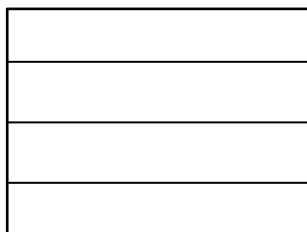
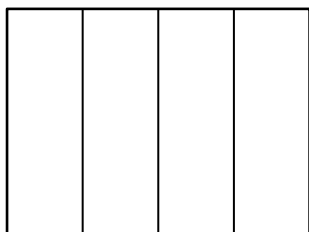
Is this a half?



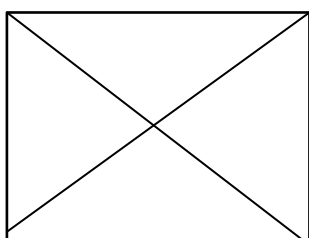
Refine definition to specify equal parts, identical parts, parts of the same size, etc.

Exploring a fourth

“Fold a piece of paper in fourths. Try to do it a different way than your neighbour is doing it. Mark the folds with a marker so we can see where you folded” Collect and display different foldings.



Challenge them to fold fourths differently.

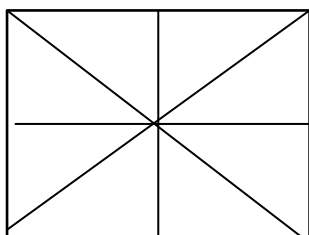


“Are these fourths?”

- Yes, they are four parts, all the same size.
- No, they are different shapes.

“Do fourths always have to be the same shape?” Refer back to rectangular foldings. Are they the same shape? How can we tell if they are all fourths if they are all different shapes?

“Are the triangular fourths the same size? Fold your paper to make triangles and try to see if they are really the same size.”



We hope someone makes new folds and argues: Each fourth triangle is made of two little triangles and they are the same shape so we know they are the same size.

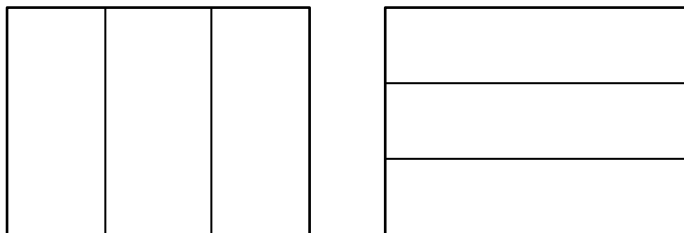
Defining a fourth

How would you define a *fourth*?”

One of four parts the same size. Write possible definitions on board.

Exploring a third

“Fold a piece of paper in thirds. Try to do it a different way than your neighbour is doing it. Mark the folds with a marker so we can see where you folded” Collect and display different foldings.



“Why are these thirds?”

- They are three parts, all the same size.

Defining a third

How would you define a *third*?”

One of three parts the same size. Write definition on board.

Defining a fraction

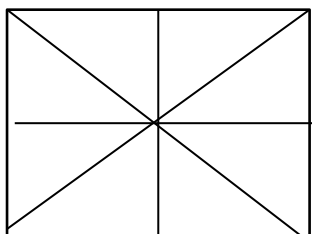
“Halves, fourths and thirds are all ‘fractions’. How would you define a fraction?”

- **One of several parts the same size.**

Write definition on board.

Notation: We write *half*, *third*, *fourth* as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$. The number below the 1 is called the ‘nevner’ because it *names* the fraction. [‘å nevne’ betyr ‘si navnet på’]. Give the children an opportunity to make connections between folded papers and the notations, discussing what the different parts of the notation means. Bring in other contexts and ask the kids to write down what they think is the fraction notation that describe the situation and explain what the ‘nevner’ means in the specific context. Go back to the folding paper context.

What fraction is this?



Comparing stambrøker

“What is the biggest, $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{4}$? What is the smallest? Make a rule for comparing fractions.

Emphasize that when comparing these ‘stambrøker’ we are referring to the same whole. Possibly arriving at:

- **The bigger the nevner the smaller the fraction.**
- **The bigger the fraction the smaller the nevner.**

Explain why your rule works for any fraction.

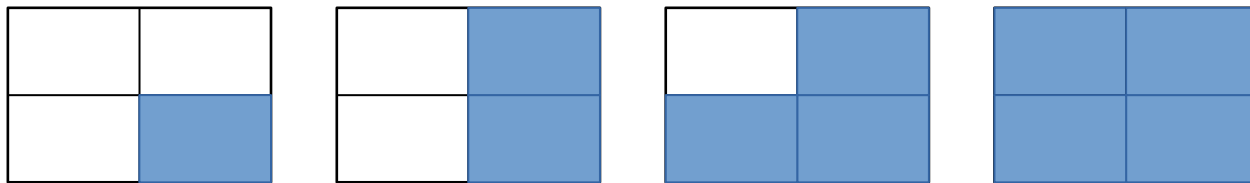
- The nevner tells how many parts the paper is divided into. The more parts the smaller they have to be.

Also explore $\frac{1}{8}$.

Lesson 2: Defining non-unit Fractions, teller, comparing fractions

Refer to folded paper from Lesson 1 throughout.

“We can count fractions. Here is 1 fourth, 2 fourths, 3 fourths and 4 fourths”



We write these $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$. The number on the top is called the ‘teller’ because it counts (det teller) how many fourths there are. Fractions with 1 as the teller we call ‘stambroker’ but the others are also fractions. How do we need to change our definition to include them?”

- One of several parts the same size. => **One or more of several parts the same size.**

Comparing fractions

(pupils working in pairs or groups)

Put the fractions in a list from smallest to biggest.

$\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{1}{2}, \frac{2}{2}, \frac{1}{1}$

(reporting)

Pair 1: Which is the smallest? How do you know? [answer should refer to rule that the bigger the nevner the smaller the fraction]

Pair 2: Which is next smallest? How do you know? [3 is next largest nevner]

Pair 3: Which is next smallest? How do you know? [Answer might be $\frac{1}{2}$ or $\frac{2}{4}$ or both. Ask if there are other answers.]

How can we show that $\frac{2}{4} = \frac{1}{2}$?

Pair 4: Which is next smallest? How do you know? [answer likely involves comparing folded parts.]

Pair 5: Which is next smallest? How do you know? [answer likely involves comparing folded parts.]

Pair 6: Which is next smallest? How do you know? [answer likely all with nevner = teller.]

How can we show that $\frac{4}{4} = \frac{3}{3} = \frac{2}{2} = \frac{1}{1}$?

Generalising

We saw that $\frac{2}{4} = \frac{1}{2}$ and $\frac{4}{4} = \frac{3}{3} = \frac{2}{2} = \frac{1}{1}$?. When will two fractions be equal? What can we say about the tellers and the nevnners?

(discussion in pairs for several minutes)

(Whole class sharing of ideas)

Possible answers that might arise.

- 1. Two fractions are equal if nevner : teller is the same.**
- 2. Two fractions are equal if nevner1 • K = nevner2 and teller1 • K = teller2.**

Write an explanation for the rule of your choice.

2. If we divide all the parts of fraction1 into K parts, then the new nevner2 is nevner1•K and the new teller teller2 is teller1•K. It is still the same fraction, we just divided each part into more parts.

Lesson 3: Folding the number line

[Materials: One metre long pieces of adding machine tape. Cutting these can be a measurement review activity for the kids.]

“We are going to look closely at the number-line. Specifically at the part of the number-line between zero and one. Label one end of your paper 0 and the other end 1.”

1. Fold the tape into two equal parts (the easy way). Label the fold $\frac{1}{2}$.
 2. Fold the tape in fourths. Start at “0” and go along the tape to the first fold. Mark it $\frac{1}{4}$. Where would two-fourths be? Mark that fold $\frac{2}{4}$. Where would three-fourths be? Mark that fold $\frac{3}{4}$.
 3. Repeat 2 for thirds, sixths, and eighths.
- Where on the tape are fractions bigger than $\frac{1}{2}$?
 - Which is bigger, $\frac{2}{3}$ or $\frac{5}{8}$? How do you know?
 - With a partner, name a fraction, and your partner should name (if possible) a fraction smaller than yours, bigger than yours, and the same as yours. If it is not possible, say why not. Take turns naming fractions.
 - Think of two fractions with the same teller. Which is closer to zero? Why?
 - Make a general rule for comparing fractions with the same teller. Why does your rule work for all fractions?

[The expected rule is: If the tellere are the same, then the bigger the nevner, the closer to 0]

Measuring Fraction strips

Measure 10cm from 0. What fraction is there? Why?

Mark the fractions at 20cm, 30cm, etc. What fraction is at 1cm? Why?

How many cm is $\frac{1}{2}$ from 0? How many tenths is $\frac{1}{2}$? Why?

How many hundredths is $\frac{1}{2}$? Why?

One half is also called '50 percent'. 'Percent' means 'av hundre' or 'hundredel'. Write 50% on the $\frac{1}{2}$ fold.

How many cm is $\frac{1}{4}$ from 0? How many hundredths is $\frac{1}{4}$?

What do you think one-fourth is called as a percent? Write that on the $\frac{1}{4}$ fold.

What percent goes on the $\frac{3}{4}$ fold? How many hundredths is $\frac{3}{4}$?

Write percents for the other fractions.

Decimals

Remember that 10cm is 0,10m.

What are the metre measures for $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{3}{4}$?

What are the metre measures for the other fractions?

How is the decimal (amount of metres) related to the percent and the number of hundredths?

Lesson 4: Parts of an amount

[Materials: 24 two-colour counters per pair.] La én av elevene I hver gruppe innta rollen som leder og sekretær, for å skrive ned det gruppa I fellesskap har kommet fram til.

1. Turn half the counters to one colour and half to the other.

- Use the counters to show that $\frac{1}{2} = \frac{2}{4}$, $\frac{1}{2} = \frac{3}{6}$, $\frac{1}{2} = \frac{4}{8}$, $\frac{1}{2} = \frac{6}{12}$ and $\frac{1}{2} = \frac{12}{24}$. Be ready to explain your answers to the class.

2. Turn one-third of the counters to one colour. How many counters are that colour? What fraction of the counters are the other colour?

- Use the counters to show that $\frac{1}{3} = \frac{2}{6}$, $\frac{1}{3} = \frac{4}{12}$, $\frac{1}{3} = \frac{8}{24}$ and $\frac{2}{3} = \frac{8}{12}$ and $\frac{2}{3} = \frac{16}{24}$. Be ready to explain your answers to the class.

3. How would you use the counters to show that $\frac{1}{4} = \frac{3}{12}$? What other fractions is $\frac{1}{4}$ equal to?

4. Gi elevene i oppdrag å lage egne oppgaver der ulike brøker skulle lages/vises - lage oppgaver til hverandre. Hvilke brøker er det mulig å lage? (Her kan noen elever påstå at man kun kan lage brøker med partall I nevner, siden de har et partall antall brikker (24). Et moteksempel her vil jo være at vi kan lage tredeler, siden 24 er delelig med 3 også)

Felles samtale: Få fram påstander. Diskutere disse. Bli enige om hvilke påstander som er riktige. Gjennomfør en metadiskusjon med klassen – hvordan kan vite ting sikkert I matematikk? Hva er egentlig et moteksempel? Hvordan kan vi argumentere matematisk for de påstandene vi har?

Punkt 2 over skal inn her.

Fra punkt 4 til det som nå heter punkt 6 skjer det et skifte I hvordan det må tenkes, fra å dele brikkene I ulike brøker til å bestemme antallet I et antall grupper til sammen. Lærer bør gjøre et poeng av dette skiftet - brøk som et antall eller del av en helhet. Vi trenger derfor punkt 5, for å gjøre overgangen til punkt 6 tydeligere.

5. Hvor mange er halvparten av de 24 brikkene? Hvor mange er $\frac{2}{4}$ av brikkene? Hvor mange er $\frac{1}{6}$ av brikkene? Hvor mange er $\frac{1}{8}$ av brikkene? Hvor mange er $\frac{1}{12}$ av brikkene?

6. How many counters is :

- a) $\frac{5}{8}$ of the counters?
- b) $\frac{3}{4}$ of the counters?
- c) $\frac{5}{6}$ of the counters?
- d) $\frac{3}{8}$ of the counters?
- e) $\frac{7}{12}$ of the counters?

7. Make a general rule for how to find out how many counters are a given fraction of a set of counters.

8. Does your rule work for any fraction and any number of counters? When does it work? Why?