Nonlinear Identification of Backlash in Robot Transmissions

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ABSTRACT

This paper addresses the issue of automatic identification of backlash in robot transmissions. Traditionally, the backlash is measured manually either by the transmission manufacturer or the robot manufacturer. Before the robot can be delivered to the end-customer, the backlash must be within specified tolerances. For robots with motor measurements only, backlash is an example of an uncontrollable behaviour which directly affects the absolute accuracy of the robot's tool-centre-point. Even if we do not attempt to bring backlash under real-time control in this paper, we will describe a method to automatically identify/estimate the backlash in the robot transmissions from torque and position measurements. Hence, only the transmissions that do not meet the backlash requirements in the automatic tests need to be checked and adjusted manually.

Keywords: Backlash, Transmission, Two-Mass Model, Friction, Nonlinear Observers, Extended Kalman Filter, Augmented state.

1 INTRODUCTION

Backlash is the shortest distance between non-driving teeth in mating gears, see Fig. 1. Backlash is difficult to estimate because it can not be described by a linear relationship. When the teeth are not in contact, the transmission force is zero. When the teeth are in contact, the force is usually proportional to the angular position difference between the gears due to elasticity.

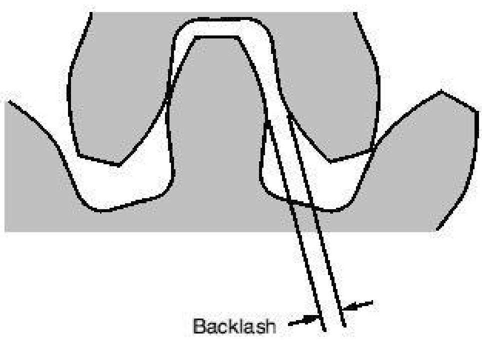

Figure 1: *Backlash in mating gear transmissions.*

The approach we propose for estimating the backlash is based on the State-Augmented Extended Kalman Filter. The original Kalman filter for linear systems emerged in the 60's and has become a mature technology for so-called "white-box" state estimation. By white-box, we mean that all parameters are known and only the state variables need to be estimated. If, however, the resulting model contains unknown parameters, it is called a gray-box model [4, 5]. The name "Extended Kalman Filter" (EKF) is used when the Kalman filter is applied to estimation of states of nonlinear dynamical systems. In this case, linearisations of the true system equations, at current estimates, are used when computing the filter gain matrices, and the filter is only an approximation of the optimal nonlinear observer (filter). The name "State Augmented Extended Kalman Filter" (SAEKF) comes from the augmentation of the (linear or nonlinear) system states $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$ with the "dynamics" $\dot{p}$ of the unknown parameters p. For instance, for constant parameters with some possible unknown external influence, the dynamics of p is often set to a "random walk" $\dot{p} = w$, where w is a stochastic noise process. What makes the augmented filter work, is the connection of the parameter estimates p and the system states x through the covariances of the noise processes.

In this paper, we report on the application of these concepts to the identification of a two-mass mechanical systems with backlash, friction and joint elasticity. The experimental work was carried out on two different robot axes.

Gray-box identification also has some advantages over black-box identification methods, such as ARMAX models or Neural Networks. Black-box models usually do not utilise prior knowledge of the system and often a large set of parameters need to be estimated. In gray-box identification, only the unknown parameters are estimated. Furthermore, if only one physical parameter changes due to a modification of the system, only that specific parameter has to be re-estimated, whereas in a black-box model, the whole set of parameters would have to be re-determined.

2 BACKLASH MODEL

The differential equations of motion for a two-mass motor-arm system are given as follows.

$$
\begin{aligned}
\dot{x_1} &= x_3 \\
\dot{x_2} &= x_4 \\
\dot{x_3} &= \frac{1}{m_1}\left(u - D(x_3 - x_4) - \tau_K - \tau_f - K_r\right) \\
\dot{x_4} &= \frac{1}{m_2}\left(D(x_3 - x_4) + \tau_K\right) \qquad (1)
\end{aligned}
$$

$$\tau_f = C_v x_3 + C_c \frac{\pi}{2} \mathrm{atan}(\alpha x_3)$$

where m_1, m_2 are the motor and arm inertias, respectively. τ_K is a nonlinear backlash and spring function to be identified. K_r is a P-controller on the motor position, u is an additional torque input used for identification, D is a known damper coefficient and τ_f is a motor friction torque. The model is written on short form as follows

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p})$$

where $p = [\tau_K]^T$ is the unknown backlash and spring (time varying) function to be estimated. τ_f is the viscous and Coloumb friction torque model. To get the EKF to work properly, a differentiable approximation of the Coloumb friction is needed (as opposed to the $\tau_f = C_c sign(\cdot)$ function) in order to compute the gradients $\frac{df}{dx}$ at speeds close to zero. The modelled friction torque is illustrated in Fig. 2.

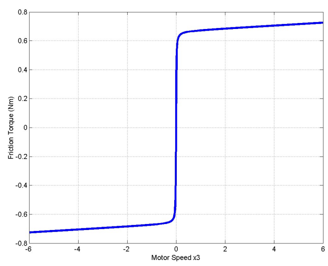

Figure 2: *Differentiable model approximation of friction torque with $C_c = 0.27$, $\alpha = 100$ and $C_v = 0.01$.*

The measurements are either motor velocity only or both motor and arm velocity. In other words, the two candidates for the measurement vector y are

$$y = x_3 + w$$

or

$$y = [x_3 + w_1, x_4 + w_2]^T,$$

where w is measurement noise. The model is illustrated in Figure 3.

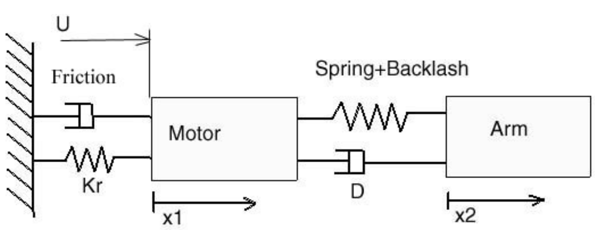

Figure 3: *Mechanical system with nonlinear backlash and motor friction.*

One advantage of the SAEKF approach, is the fact that we do not have to assume a particular model structure for the backlash and spring function to be identified. Instead, we make τ_K an augmented state. This feature will become clearer in the following section.

3 AUGMENTED STATES

Traditionally, in gray box identification, functions depending on some tuning parameters have been used to approximate nonlinear relationships. These constant parameters are then tuned using the measurements. As an example, a 3rd order polynomial $y = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3$ is shown in Figure 4 for the approximation of a linear spring with backlash. This "parametric approach" for approximating backlash has several drawbacks:

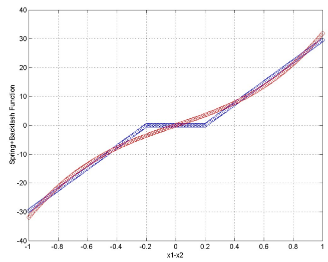

Figure 4: *Polynomial approximation to backlash and spring function.*

- The polynomial obtained is a differentiable function and it becomes difficult to estimate the size of the backlash from the coefficients $\alpha_0, \alpha_1, \alpha_2, \alpha_3$.
- The polynomial approach introduces several parameters, in this case four, which increases the complexity of the problem.

What makes the SAEKF approach really useful is the fact that the parameters to be estimated do not have to be constants, but can be time varying or depend in a nonlinear fashion on the current state. The main new idea of this paper is to replace the polynomial coefficients with only one augmented state for the spring and backlash force τ_K. The augmented state filter will then estimate the force τ_K as a function of time and no particular shape of the underlying nonlinearity has to be assumed.

Augmented states are introduced into the system equations in the following way.

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p}) \\ 0 \end{bmatrix} + v \quad (2)$$

where v is certain process noise with zero mean. In our concrete two-mass problem the complete augmented model boils down to:

$$\begin{aligned} \dot{x_1} &= x_3 + v_1 \\ \dot{x_2} &= x_4 + v_2 \\ \dot{x_3} &= \frac{1}{m_1}(u - D(x_3 - x_4) - p - \tau_f - K_r) + v_3 \\ \dot{x_4} &= \frac{1}{m_2}(D(x_3 - x_4) + p) + v_4 \\ \dot{p} &= v_5 \end{aligned}$$

where v_i variables are noise processes acting on the state and parameter variables. The covariance matrix for the augmented noise process v is given as

$$E(v(t)v^T(t')) = \begin{bmatrix} \mathbf{Q}_{xx} & 0 \\ 0 & \mathbf{Q}_{pp} \end{bmatrix} \delta(t-t') \qquad (3)$$

where δ is the unit impulse (Dirac) function, $\mathbf{Q}_{xx}$ is the covariance of the noise on the state derivatives. Since the position states are simply derivatives of the velocity states, the covariances of the noise processes v_1 and v_2 can be set to zero, ie. $\mathbf{Q}_{xx}(1,1) = 0$ and $\mathbf{Q}_{xx}(2,2) = 0$. $\mathbf{Q}_{pp}$ is the covariance of the noise on the augmented state derivative, i.e. the "time derivative" of the backlash and spring force. $\mathbf{Q}_{pp}$ can be seen as the main tuning parameter of the augmented filter. If $\mathbf{Q}_{pp}$ is set to zero, the parameter p will remain a constant and equal to its initial value. The larger the value of $\mathbf{Q}_{pp}$, the faster the augmented state will be updated as new measurements come in, but also the larger will be the variance of the estimates.

Note that for nonlinear systems, even if the disturbances were Gaussian and white, and the covariances $\mathbf{Q}_{xx}$ and $\mathbf{R}$ were known (a rather unrealistic assumption), their true values would not necessarily give the best filter performance. This occurs because the propagation and update equations of the filter are approximations. An increased value of the covariance matrices can reduce the effects of these approximations, [6].

4 AUGMENTED FILTER EQUATIONS

The augmented filter equations were presented in [1] and we summarise them here in Table 1 for completeness. Note that the equations for the state and covariance propagation in Table 1 are a mixture of continuous and discrete equations. First, using the nonlinear system model and modified Riccati equations, the state vector and the covariances are propagated continuously in $t_k \leq t \leq t_{k+1}^-$. Secondly, the state and parameter vectors and the covariances are updated using Kalman filter gains computed via the predictions made in the previous step. Note how the quadratic terms of the Riccati equations are introduced in the covariance update step and not in the predictor step. This is done mainly for numerical stability. The nonlinear system is only linearised (at the current estimates at time t_k) when computing the covariance updates and the filter gain coefficients. For the state estimate propagation, the complete nonlinear model is used.

5 SIMULATIONS 1

In this section we present some simulation results for the Augmented filter. Although the position and velocity states are simulated, the input torque in Fig. 5 and the mass values are representative for an ABB industrial type robot.

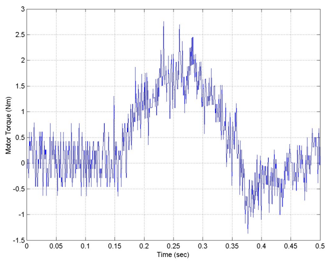

Figure 5: *Motor torque input used for identification of backlash and spring function.*

The model parameters were set as follows.

$$\begin{aligned} m_1 &= 0.006442 \\ m_2 &= 0.023454 \\ D &= 0.01 \\ K_r &= 1 \\ C_v &= 0.2 \\ C_c &= 0 \end{aligned}$$

The covariance matrices were set as follows.

$$\mathbf{Q}_{pp} = 1 \quad \mathbf{R} = 2.5 \cdot 10^{-4}$$

$$\mathbf{Q}_{xx} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\mathbf{R}$ is the covariance of the measurement (motor and/or arm velocity) noise w and was a calculated value. The other parameters, $\mathbf{Q}_{pp}$ and $\mathbf{Q}_{xx}$ were tuned manually. Note that a noise on the input signal u can be expressed as part of $\mathbf{Q}_{xx}$. A noise signal v_u on u is equivalent to a noise signal $v_3 = \frac{v_u}{m_1}$ on the acceleration $\dot{x}_3$. Hence, noise on the input torque can be modelled by element $\mathbf{Q}_{xx}(3,3)$.

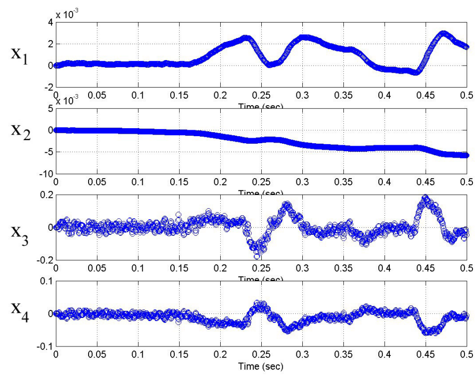

Figure 6: *Model state predictions with measurements of motor velocity only.*

Fig. 6 shows the position and velocity states in the order (x_1, x_2, x_3, x_4). The state x_3 was the only measure-

State estimate propagation $t_k \le t \le t_{k+1}^-$	$\dot{\hat{\mathbf{x}}} = \mathbf{f}(\hat{\mathbf{x}}, \mathbf{u}, \hat{\mathbf{p}}(t_k))$
Covariance propagation $t_k \le t \le t_{k+1}^-$	$\dot{\mathbf{P}}_{xx} = \frac{d\mathbf{f}}{d\mathbf{x}^T}\mathbf{P}_{xx} + \mathbf{P}_{xx}\frac{d\mathbf{f}^T}{d\mathbf{x}} + \frac{d\mathbf{f}}{d\mathbf{p}^T}\mathbf{P}_{xp}^T + \mathbf{P}_{xp}\frac{d\mathbf{f}^T}{d\mathbf{p}} + \mathbf{Q}_{xx}$ $\dot{\mathbf{P}}_{xp} = \frac{d\mathbf{f}}{d\mathbf{x}^T}\mathbf{P}_{xp} + \frac{d\mathbf{f}}{d\mathbf{p}^T}\mathbf{P}_{pp}$ $\dot{\mathbf{P}}_{pp} = \mathbf{Q}_{pp}$
Predicted output	$\hat{\mathbf{y}}(t_{k+1}) = \mathbf{h}(\hat{\mathbf{x}}(t_{k+1}^-), \mathbf{u}(t_{k+1}), \hat{\mathbf{p}}(t_k))$
Prediction error	$\mathbf{e}(t_{k+1}) = \mathbf{y}(t_{k+1}) - \hat{\mathbf{y}}(t_{k+1})$
Approximate prediction error covariance matrix	$\mathbf{A}(t_{k+1}) = \mathbf{R} + \frac{d\mathbf{h}}{d\mathbf{x}^T}\mathbf{P}_{xx}(t_{k+1}^-)\frac{d\mathbf{h}^T}{d\mathbf{x}} + \frac{d\mathbf{h}}{d\mathbf{x}^T}\mathbf{P}_{xp}(t_{k+1}^-)\frac{d\mathbf{h}^T}{d\mathbf{p}}$ $+\frac{d\mathbf{h}}{d\mathbf{p}^T}\mathbf{P}_{xp}^T(t_{k+1}^-)\frac{d\mathbf{h}^T}{d\mathbf{x}} + \frac{d\mathbf{h}}{d\mathbf{p}^T}\mathbf{P}_{pp}(t_{k+1}^-)\frac{d\mathbf{h}^T}{d\mathbf{p}}$
Filter gain matrices	$\mathbf{K}(t_{k+1}) = \left(\mathbf{P}_{xx}(t_{k+1}^-)\frac{d\mathbf{h}^T}{d\mathbf{x}} + \mathbf{P}_{xp}(t_{k+1}^-)\frac{d\mathbf{h}^T}{d\mathbf{p}}\right)\mathbf{A}^{-1}(t_{k+1})$ $\mathbf{L}(t_{k+1}) = \left(\mathbf{P}_{xp}^T(t_{k+1}^-)\frac{d\mathbf{h}^T}{d\mathbf{x}} + \mathbf{P}_{pp}(t_{k+1}^-)\frac{d\mathbf{h}^T}{d\mathbf{p}}\right)\mathbf{A}^{-1}(t_{k+1})$
State update	$\hat{\mathbf{x}}(t_{k+1}) = \hat{\mathbf{x}}(t_{k+1}^-) + \mathbf{K}(t_{k+1})\mathbf{e}(t_{k+1})$
Parameter update	$\hat{\mathbf{p}}(t_{k+1}) = \hat{\mathbf{p}}(t_{k+1}^-) + \mathbf{L}(t_{k+1})\mathbf{e}(t_{k+1})$
Covariance update	$\mathbf{P}_{xx}(t_{k+1}) = \mathbf{P}_{xx}(t_{k+1}^-) - \mathbf{K}(t_{k+1})\mathbf{A}(t_{k+1})\mathbf{K}^T(t_{k+1})$ $\mathbf{P}_{xp}(t_{k+1}^+) = \mathbf{P}_{xp}(t_{k+1}^-) - \mathbf{K}(t_{k+1})\mathbf{A}(t_{k+1})\mathbf{L}^T(t_{k+1})$ $\mathbf{P}_{pp}(t_{k+1}^+) = \mathbf{P}_{pp}(t_{k+1}^-) - \mathbf{L}(t_{k+1})\mathbf{A}(t_{k+1})\mathbf{L}^T(t_{k+1})$

Table 1: *State-augmented Extended Kalman Filter (SAEKF) .*

ment, ie

$$\mathbf{y} = h(\mathbf{x}, \mathbf{u}, \mathbf{p}) = x_3 + w \tag{4}$$

The other states x_1, x_2 and x_4 were simulated/estimated by the Kalman filter.

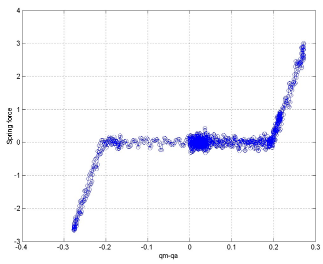

Figure 7: *Estimate of backlash function τ_K with measurements of motor velocity only. The true backlash is* 0.2 *rad.*

Fig. 7 shows the simulated augmented state τ_K as a function of the position difference $x_1 - x_2$. The true backlash and spring function for τ_K had a backlash of 0.2 rad and a spring coefficient of $37\frac{Nm}{rad}$. Note how well the augmented state describes the nonlinearity. We stress the fact that no apriori assumptions on the shape of the nonlinearity were made.

Fig. 8 shows the augmented state as a function of time. We clearly see the benefits of the SAEKF approach: the augmented state is able to track rapid changes and can be used to model time-varying parameters.

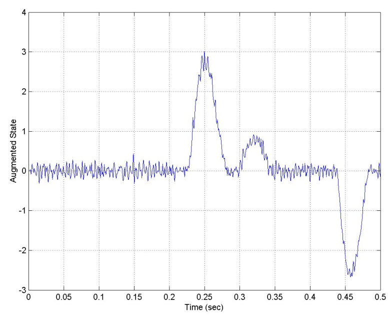

Figure 8: *Augmented state as a function of time .*

The real motor velocity resulting from the input torque in Fig. 5 is shown in Fig. 9. Clearly, this measurement on the real robot does not match the modelled motor velocity x_3 in Fig. 6. In this case, the SAEKF is also not able to produce sensible estimates of the backlash and spring force. It follows that the masses, dampers and friction parameters must be accurately identified prior to making the nonlinear identification experiments. Alternatively, they can/must be estimated together with the backlash using the same SAEKF ideas. Possible approaches for separately estimating rigid-body parameters and friction are described in [2, 3].

6 SIMULATIONS 2

A similar simulation was made on a different ABB robot model, but this time accurate rigid-body param-

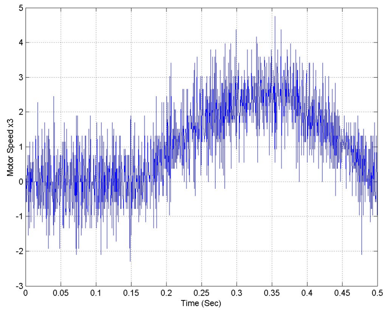

Figure 9: *Real measured motor velocity resulting from torque input in Fig. 5.*

eters were available. Further, we used a more aggressive excitation signal (compare the excitations in Figs. 5 and 10) and measurements of both motor and arm velocities. As for Section 5, we first simulated the measurements using a realistic motor torque input. A PRBS motor torque signal is shown (for $0 \leq t \leq 0.045$) in Fig. 10. The model parameters were set as follows

$$\begin{aligned} m_1 &= 0.000800 \\ m_2 &= 0.001765 \\ D &= 0.01, K_r = 0 \\ C_c &= 0.27, C_v = 0.01, \alpha = 100 \end{aligned}$$

and for the covariance matrices

$$\begin{aligned} \mathbf{Q}_{pp} &= 100, \quad \mathbf{R} = 1.0 \\ \mathbf{Q}_{xx} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

Note that the controller K_r is set to zero in these tests,

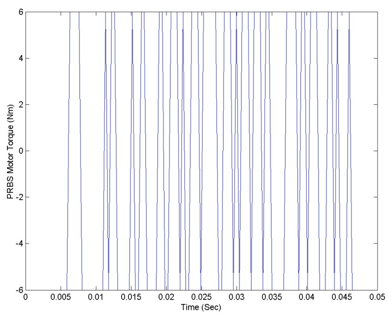

Figure 10: *Piecewise random binary signal (PRBS) used as motor torque input.*

while in the previous section K_r was equal to one. For the control system used in the first test, the controller could not be switched off for safety reasons, while the tests in this section were made with a prototype controller without these safety requirements.

Both for the simulations in this section and with the real measurements in the following section, we experienced

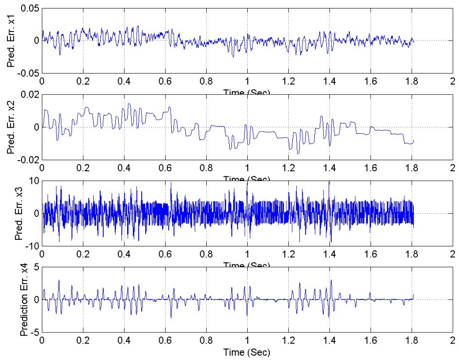

Figure 11: *Model state prediction errors with simulated measurements of both motor and arm velocities.*

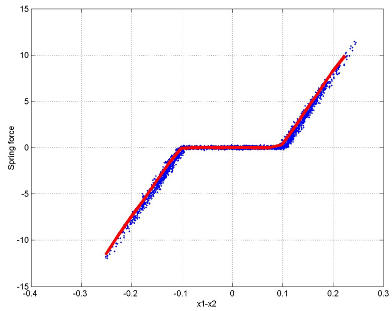

Figure 12: *Estimate of backlash function τ_K with simulated measurements of both motor and arm velocities. The true backlash is 0.1 rad and the true spring constant is $79\frac{Nm}{rad}$.*

a significant improvement in the filter performance by increasing the covariance matrices, as suggested by [6].

Fig. 11 shows model state prediction errors found by comparing the simulated states with those of the Augmented filter. The filter does a good job and produces zero mean residuals. Fig. 12 shows the identified nonlinear backlash and spring function vs. the motor and arm position difference. In this case, the true backlash size was set to 0.1 rad. The solid line in the figure is the mean value of the augmented state (τ_K) and is a very good description of the true backlash and spring function.

7 SIMULATIONS 3

In this section we repeat the test in the previous section, but with real measurements of motor and arm velocity. This time we double-checked that the simulated measurements were a reasonable match to the real measurements and we confirmed that the two-mass model with the rigid-body and friction parameters from the previous section is a good description of the real robot axis. Since the transmission model is unknown, we can only make an assumption on the transmission force in the simulations and a rough comparison between the simulated model and the real measurements.

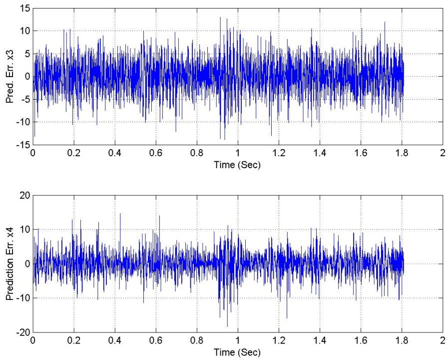

Figure 13: *Measurement prediction errors with real measurements of both motor and arm velocities.*

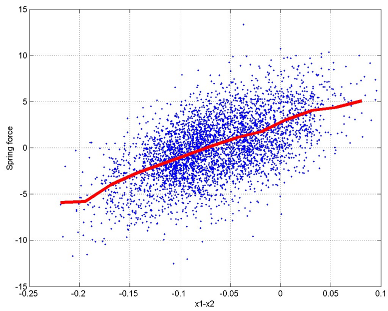

Figure 14: *Estimate of backlash function τ_K with real measurements of both motor and arm velocities.*

Fig. 13 shows the differences between the measurements and the corresponding filter predictions. The filter produces residuals with zero mean as desired. Fig. 14 shows the estimate of τ_K. We see that there is no evidence of backlash for this particular robot axis. We believe that the spread of the augmented values is caused by the fact that the two-mass model with the chosen mass and friction parameters is not an exact description of the real system.

The mean values of the transmission force behave like a linear spring with coefficient $K \approx 67\frac{Nm}{rad}$. Again, we see the benefit of not making an assumption on the shape of the nonlinearity. If the transmission model happens to be linear, the filter will still give us the correct answer! In fact, the experiments in Fig. 14 were made on a new ABB robot model with completely backlash-free transmissions.

8 CONCLUSIONS

This paper has presented an approach for estimating a nonlinear transmission force by using the Augmented State Extended Kalman Filter approach. The main new idea of the paper is to replace the common approach of approximations via polynomials by the estimation of a single augmented state. This significantly reduces the complexity of the problem and makes possible to track reasonably fast model dynamics. Another advantage of the single augmented state approach, is that no assumptions on the nonlinear shape has to be made. At least in principle, the augmented τ_K is able to track any kind of nonlinearity, including complex hysteresis functions with memory.

However, there are several considerations which have to be made before a good estimate is obtained. First, all model parameters other than the transmission model must be well known or estimated, before hand or simultaneously. Without a good model description, the filter is not able to produce reasonable estimates of the transmission force. In other words, robustness of the filter with respect to model and parameter errors is a crucial issue. Second, the covariance matrices $\mathbf{Q}_{xx}$, $\mathbf{Q}_{pp}$ and $\mathbf{R}$ must be carefully tuned. As suggested by [6], increased values of the covariances can increase the filter performance.

In this particular application, a good test to detect modelling errors is to simulate the measurements by using the real input signal and compare with the real measurements. Further, the input signal must be aggressive enough to create persistent excitation of the transmission force. Ideally, one should avoid going through the backlash region at the same time as going through the zero motor velocity region where the static friction force has a large influence. In other words, the motor-arm difference $x_1 - x_2$ should be outside the backlash region when the motor velocity x_3 changes direction.

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