UNIVERSITETET I AGDER

Guest lecture:

Nonlinear damping in system dynamics and control

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03.11.2020
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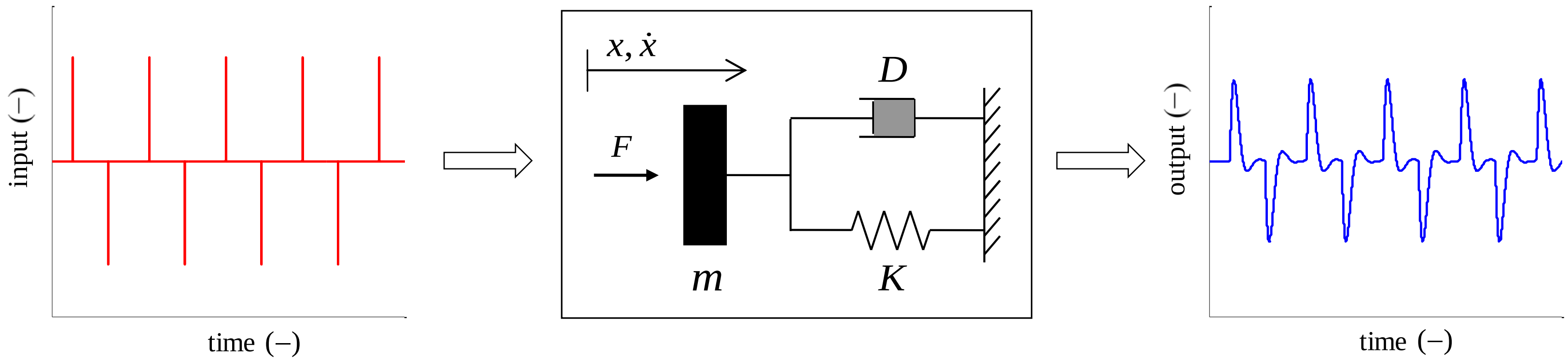
Outline

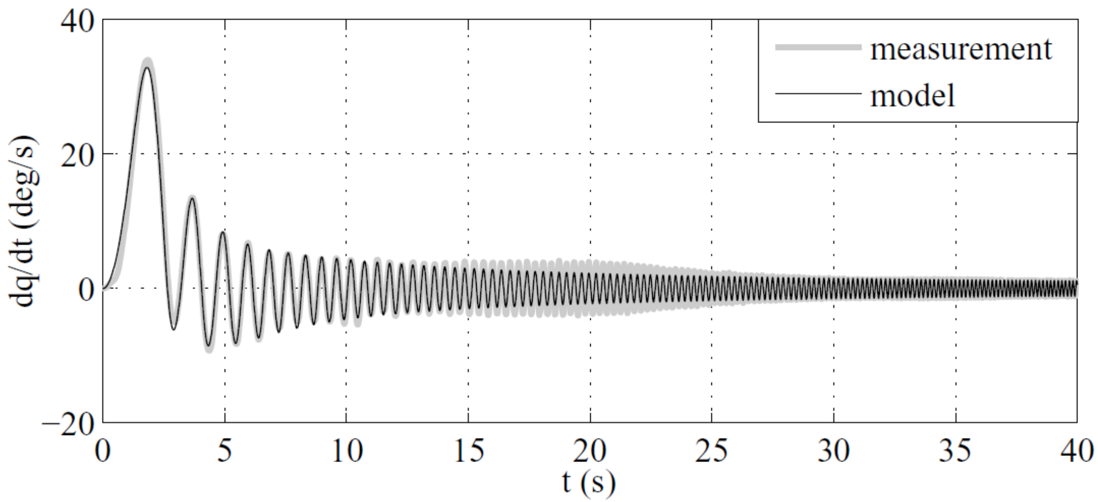
- Nonlinear damping in dynamic (motion) systems
- Optimal nonlinear damping control for 2nd order systems

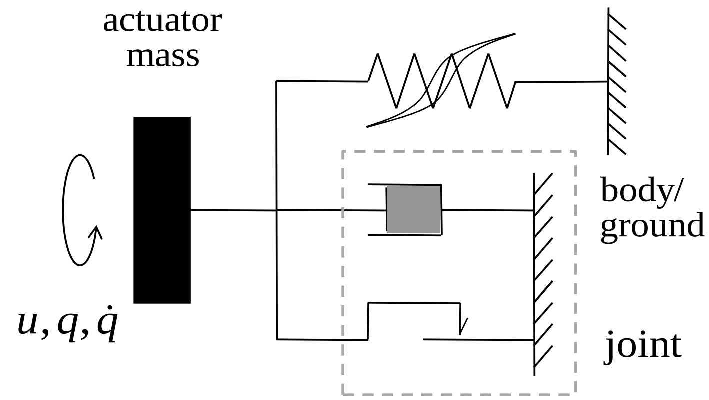
- Actuator model as a second-order linear system

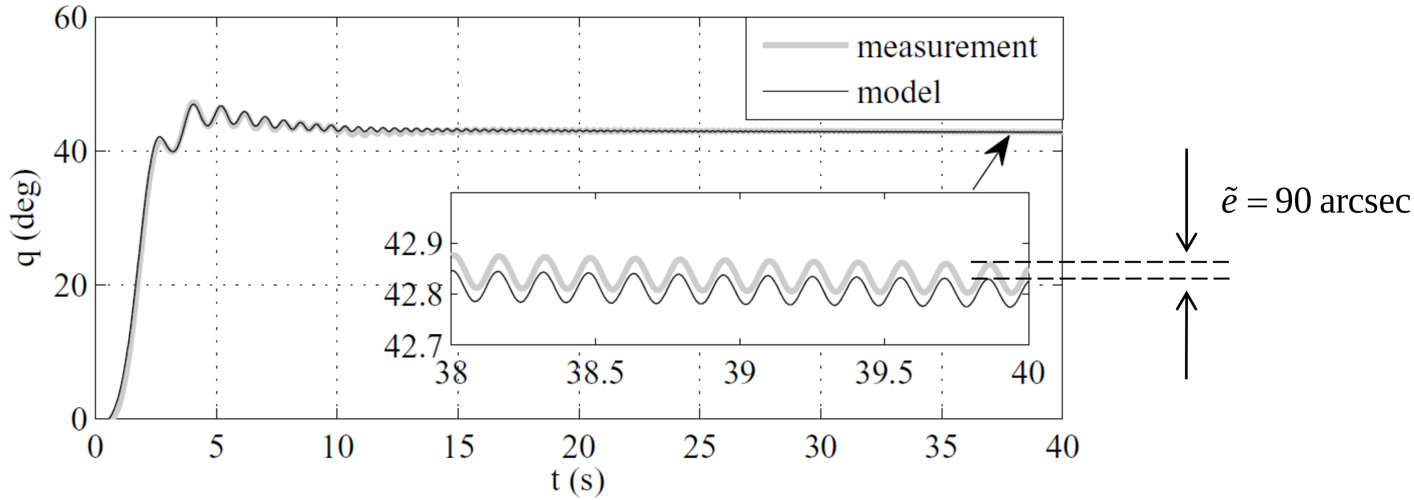


- Example of a real physical (actuator) system

Starting from simple..., cont.







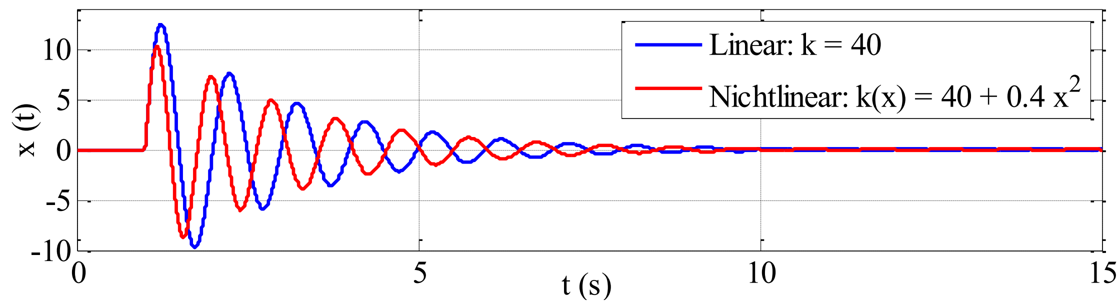
Starting from simple…, cont.

- Second-order (damped) oscillating dynamics, when linear…

$$m\ddot{x} + d\dot{x} + kx = 0$$

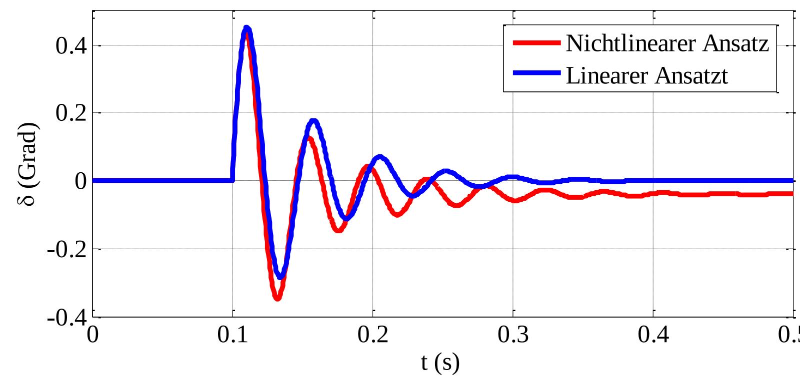
$$\omega_d = \omega_0\sqrt{1-\xi^2} = \sqrt{\frac{k}{m} - \frac{d^2}{4m^2}} = \text{const}$$

- Free/eigen response (cf. with hardening *k*)



- More general nonlinear response

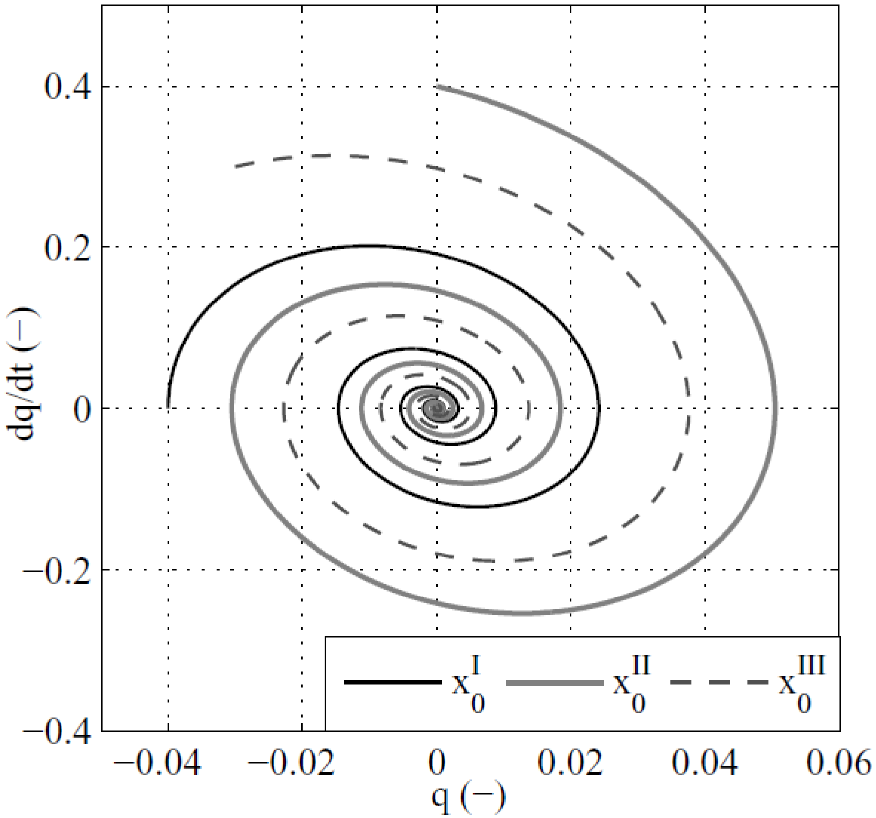
$$m\ddot{x} + \sigma(x, \dot{x}) + h(x) = 0$$



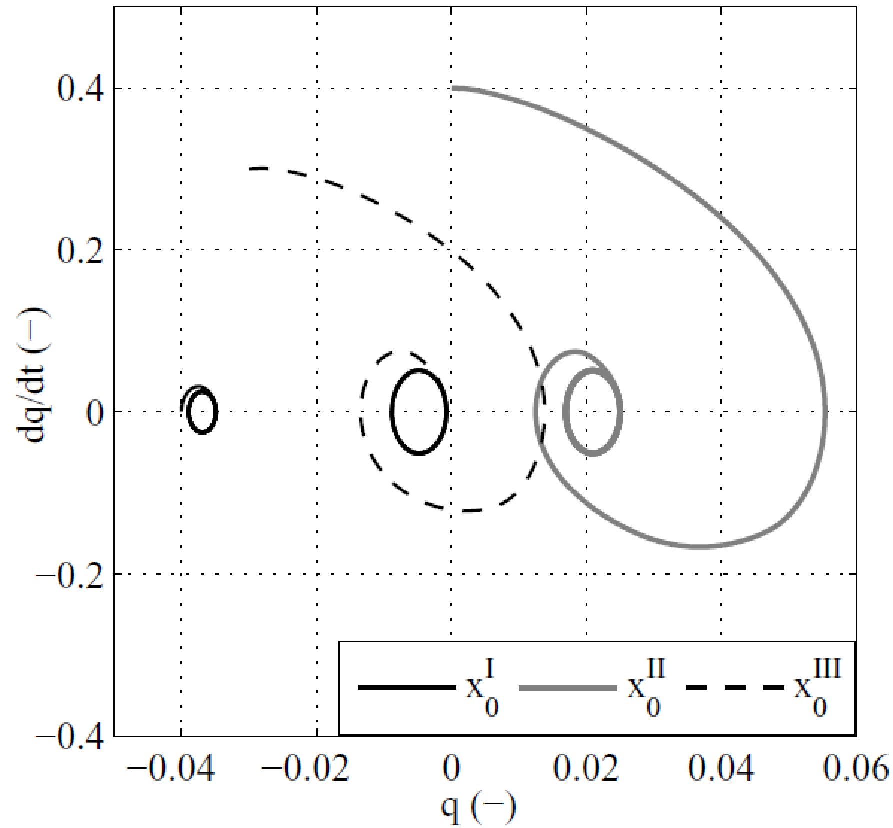
Starting from simple..., cont.

- Impact on the equilibrium (cf. linear & nonlinear in phase-plane)

Linear system

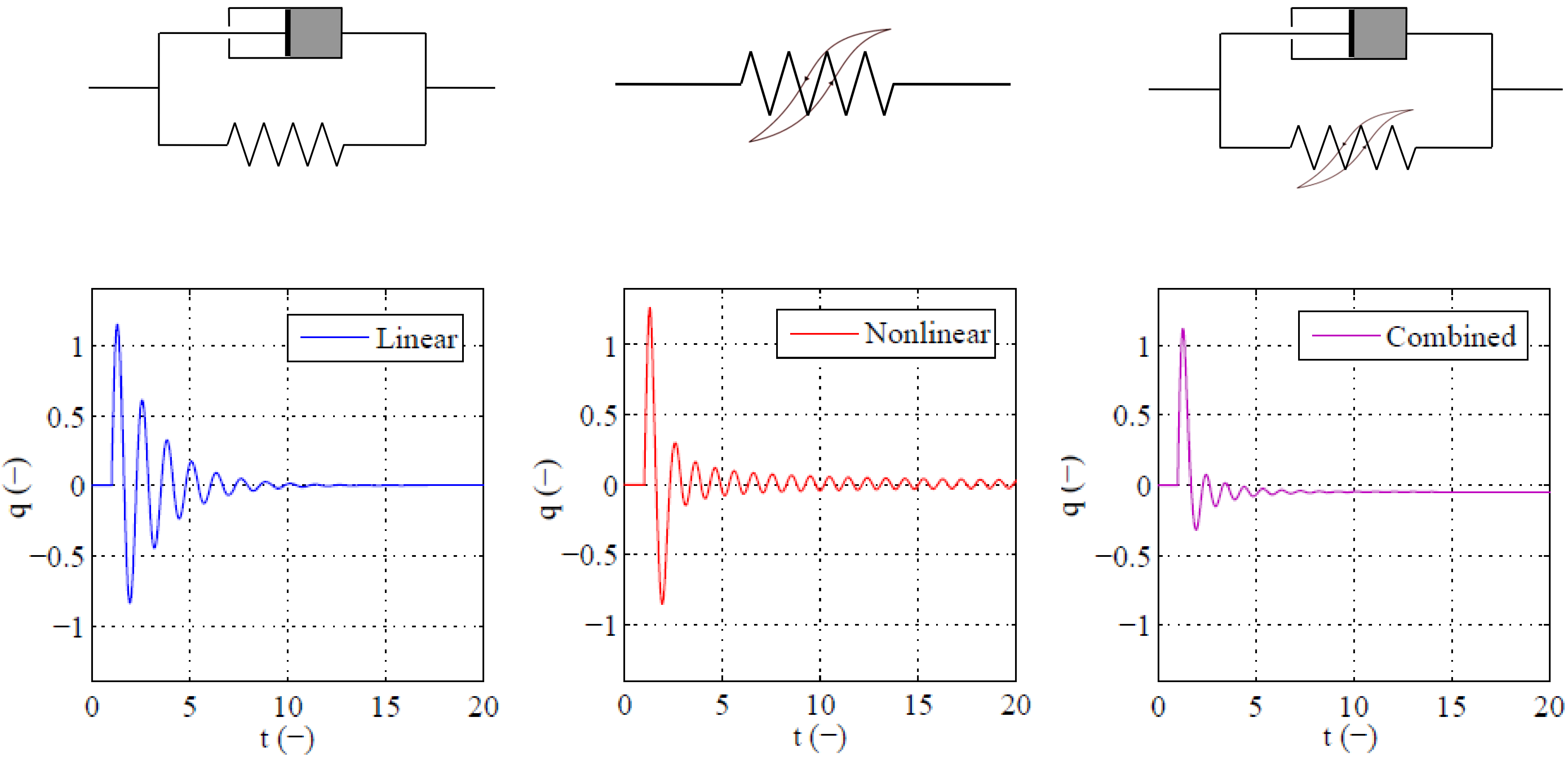


Nonlinear system with memory



Starting from simple..., cont.

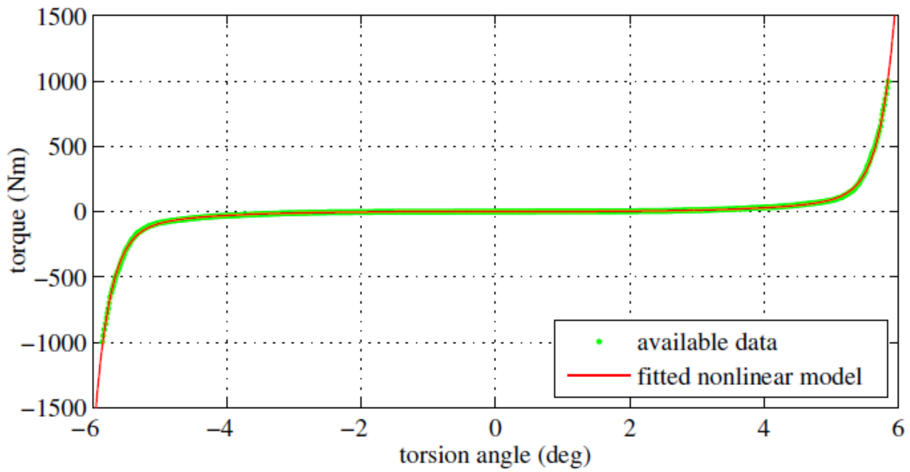
- Stiffness-damping structures

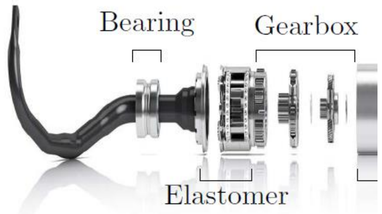


Motivating practical observations

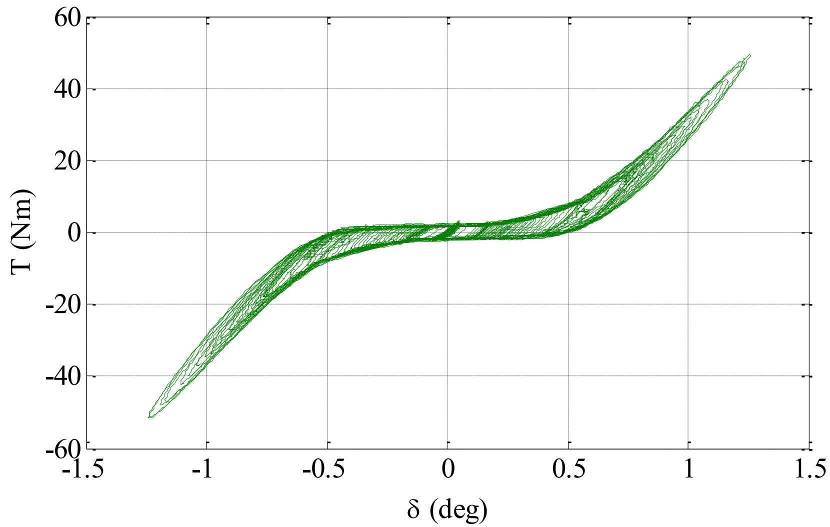
- Nonlinear stiffness and nonlinear stiffness coupled with damping

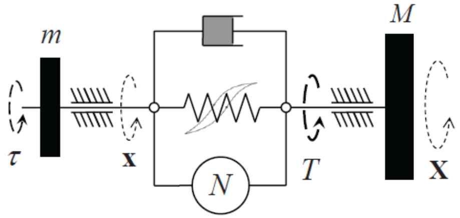
Elastomer decoupling unit from active antiroll bar (measurements)





Gears with elasticities and coupling in motor-drive systems (measurements)

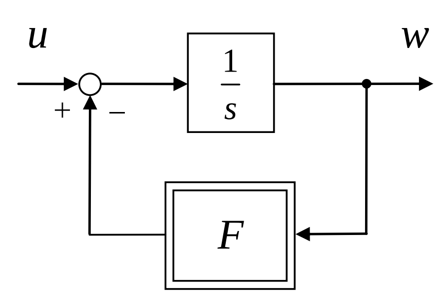


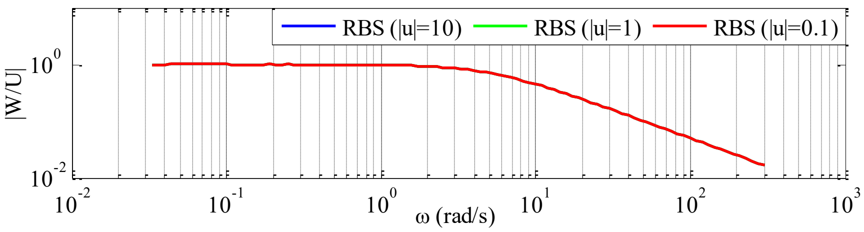


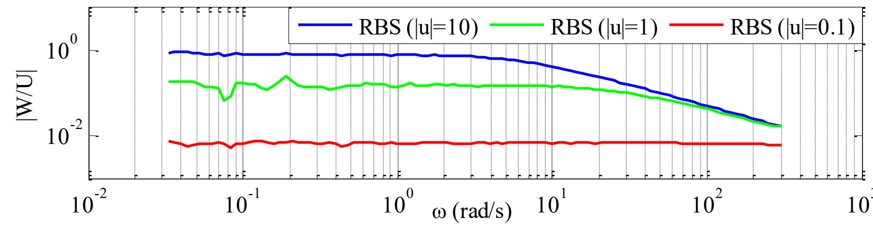
Motivating practical observations, cont.

- Nonlinear frictional damping

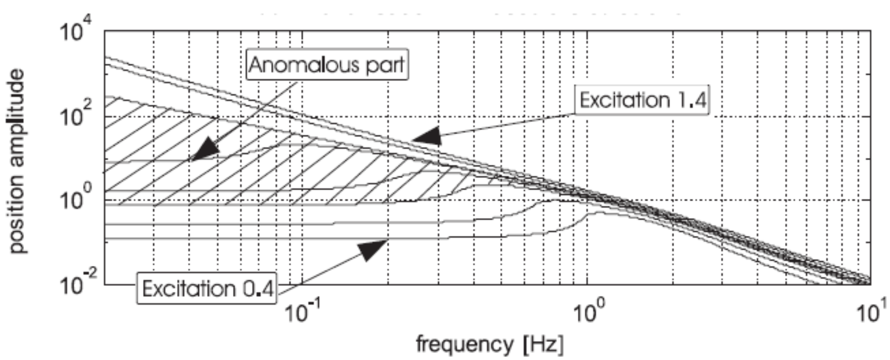
$$\dot{w}(t) + F\big(w(t)\big) = u(t)$$



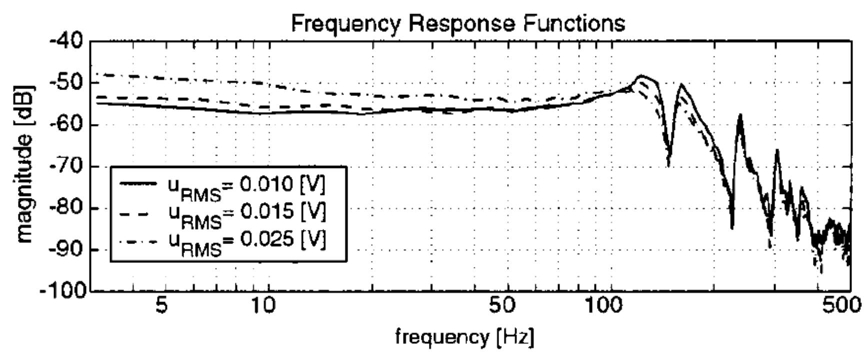




- Some measured amplitude-dependent FRFs

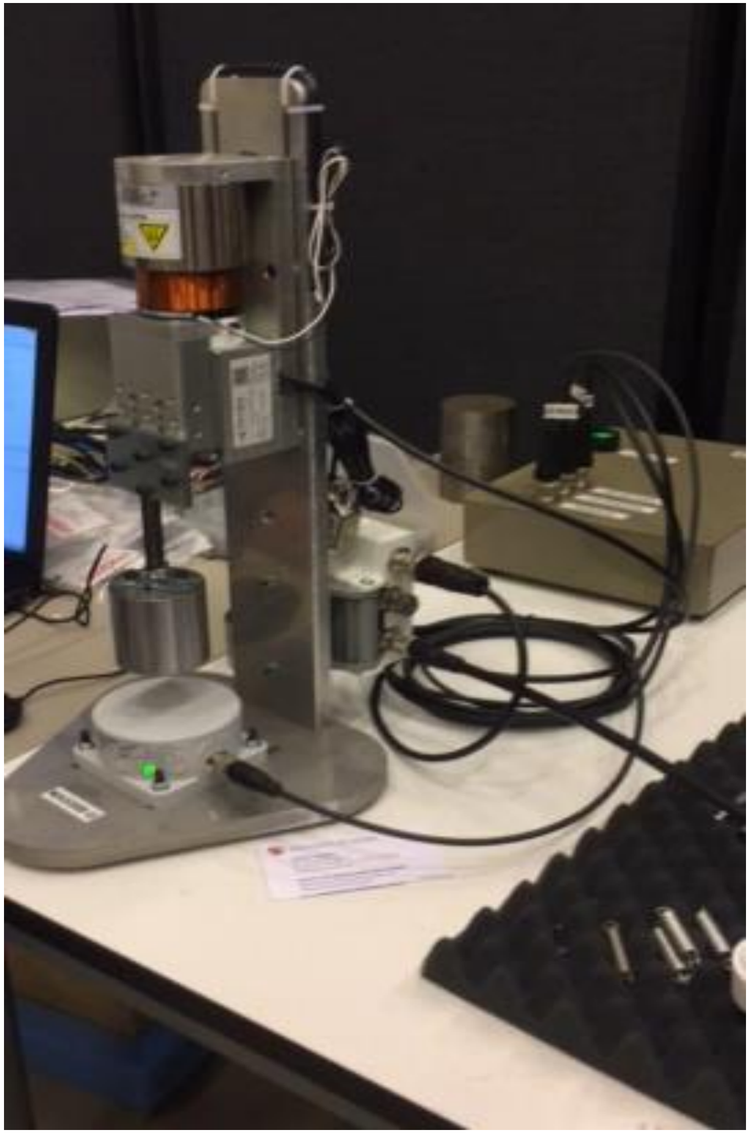


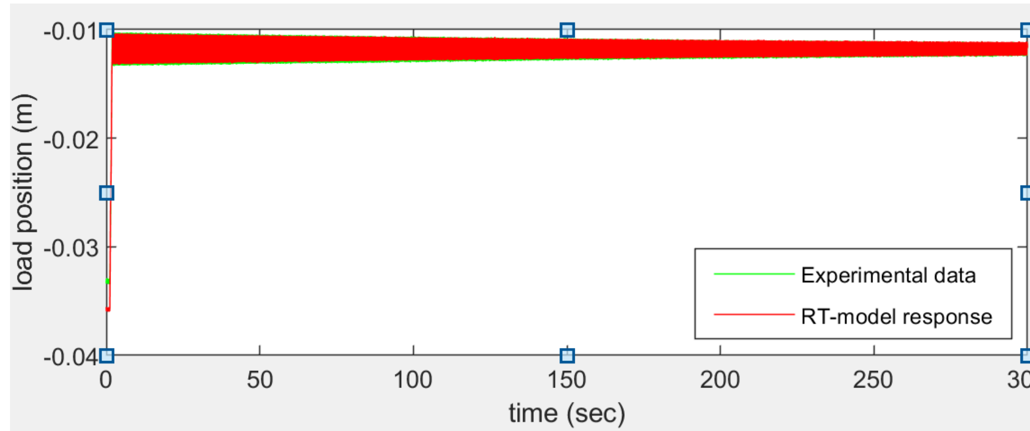
Theoretical analysis of the dynamic behavior of hysteresis elements in mechanical systems, F. Al-Bender, W. Symens, J. Swevers, H. Van Brussel, Int. J. of Non-Linear Mechanics, 2004

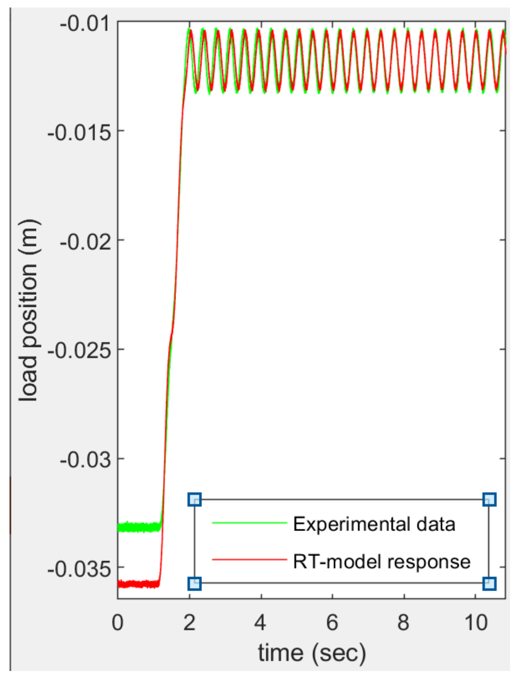


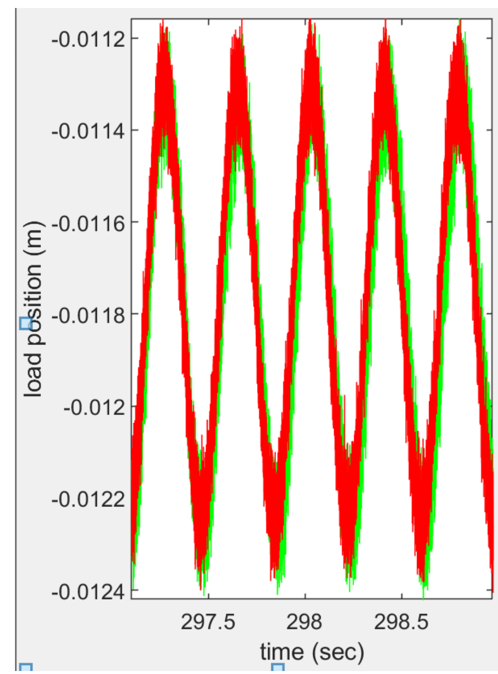
Frequency domain identification of dynamic friction model parameters, R.H.A. Hensen; M. van de Molengraft; M. Steinbuch, IEEE Trans. on Con. Sys. Tech., 2002

Motivating practical observations, cont.



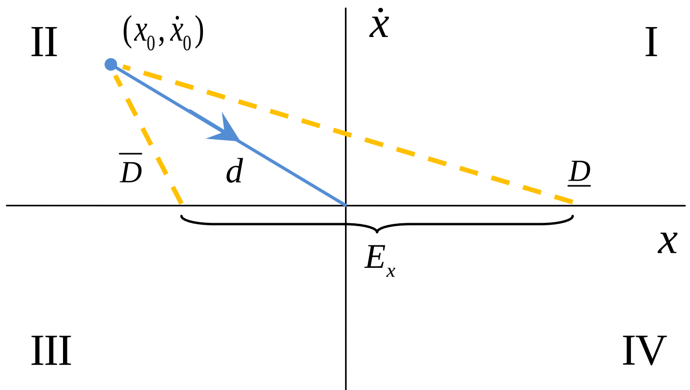




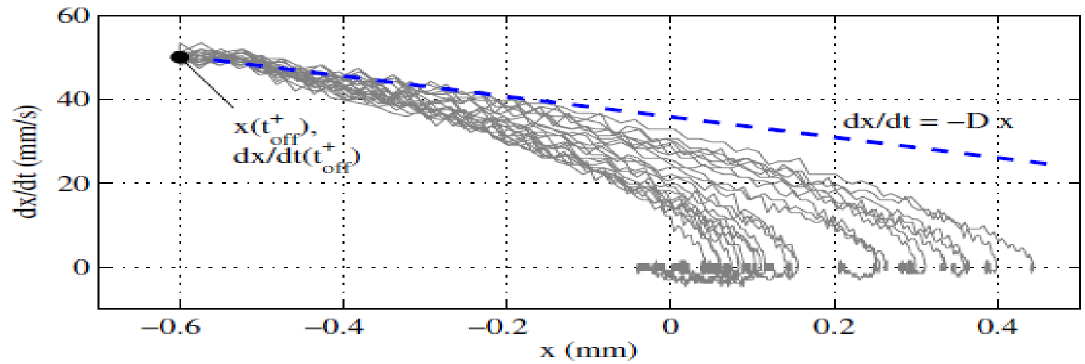


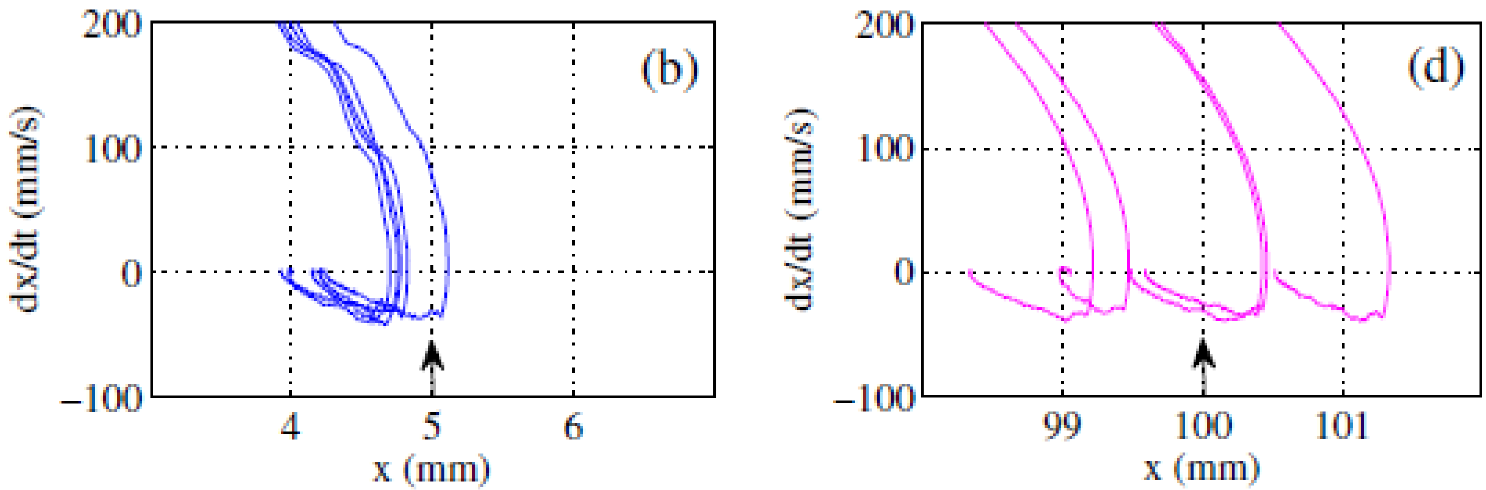
Motivating practical observations, cont.

- Motion trajectories with uncertain and nonlinear damping



Infinite set of equilibria E_x

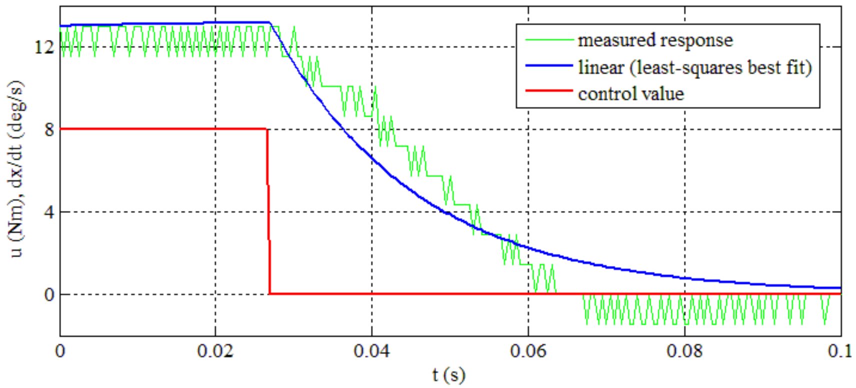




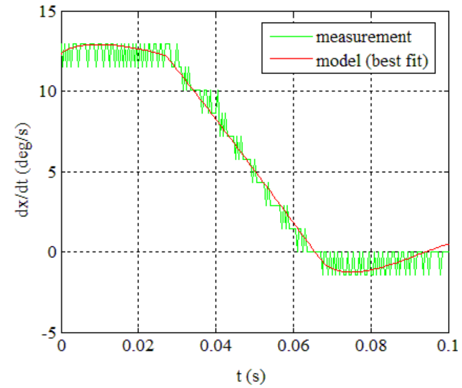
Motivating practical observations, cont.

- Convergence is vicinity to the motion stop

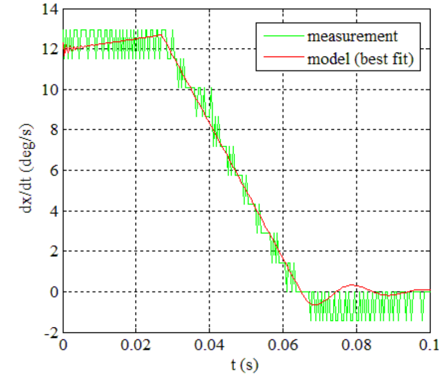
Linear (viscous) friction damping



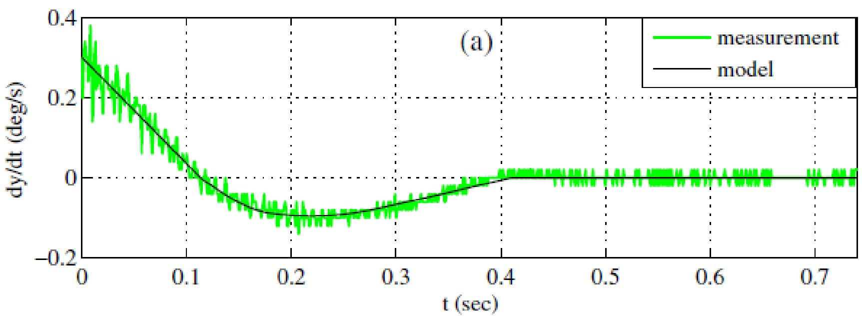
Dahl friction

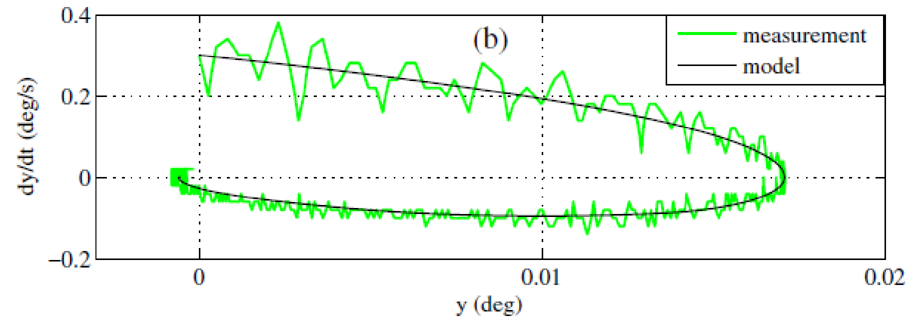


LuGre friction



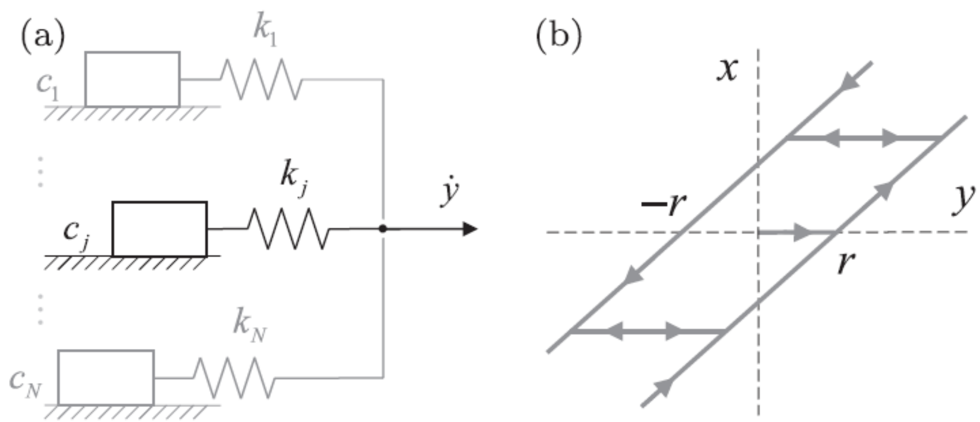
- More advanced (rheology-based) friction, *e.g. Prandtl-Ishlinskii type*





Nonlinear damping by Prandtl-Ishlinskii operator

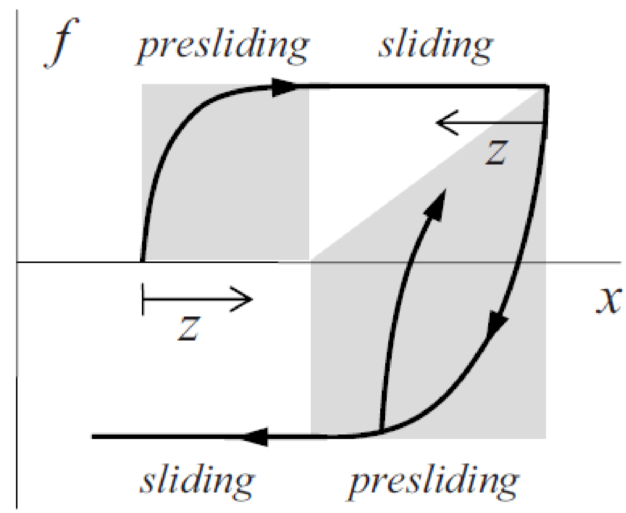
- Prandtl-Ishlinskii (PI) model ≡ Maxwell-slip (MS) model



$$\dot{x} = \begin{cases} \dot{y} & \text{if } x - r = y \vee x + r = y \\ 0 & \text{if } x - r < y < x + r. \end{cases}$$

$$F_j = k_j\big(y - x(y, r_j)\big)$$

$$F(\dot{y}) = \sum_{j=1}^{N} F_j$$



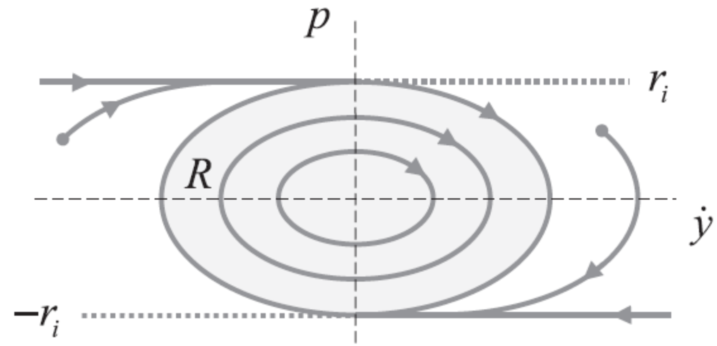
Presliding hysteresis damping of LuGre and Maxwell-slip friction models,
M Ruderman, Mechatronics 30, pp. 225-230, 2015

Nonlinear damping by PI operator, cont.

- First, consider motion dynamics with only one i-th PI-element

$$\ddot{y} = -k_i y + k_i x(y, r_i)$$

$$p = x - y \qquad -r_i \leqslant p \leqslant r_i$$



$$V = \frac{1}{2k_i}\dot{y}^2 + \frac{1}{2}p^2$$

$$V \to \infty \text{ as } \|(\dot{y}, p)\| \to \infty$$

$$\dot{V} = \dot{y}p + \dot{p}p \qquad \dot{V} = \begin{cases} 0 & \text{within a play-zone, i.e. at } \dot{p} = -\dot{y} \\ \dot{y}p & \text{otherwise, i.e. at } \dot{p} = 0. \end{cases}$$

For second case, one can show that $\dot{y} > 0$ implies $p = -r_i$ and, correspondingly, $\dot{y} < 0$ implies $p = r_i$. Thus, $\dot{V} < 0$ whenever the PI operator is not within the play-zone.

Nonlinear damping by PI operator, cont.

- Motion dynamics with entire PI operator (*N* elements, evt. $N \to \infty$)

$$\ddot{y} = -\mathbf{k}^T \mathbf{y} + \mathbf{k}^T \mathbf{x}(y, \mathbf{r}) \qquad V = \frac{1}{2}\dot{y}^2 + \frac{1}{2}\mathbf{p}^T \mathbf{K} \mathbf{p}$$

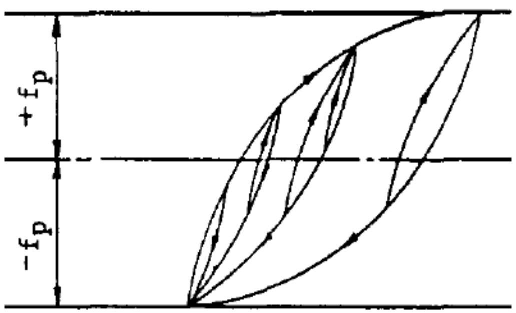
$$\dot{V} = \dot{y}\mathbf{k}^T \mathbf{p} + \dot{\mathbf{p}}^T \mathbf{K} \mathbf{p} \qquad \dot{V} = \begin{cases} \dot{y}\mathbf{k}^T \mathbf{p} - \dot{\mathbf{y}}^T \mathbf{K} \mathbf{p} = 0 & \text{if } \forall j \in N \; \dot{\mathbf{p}}(j) = -\dot{y}, \\ \dot{y}\mathbf{k}^T \mathbf{p} < 0 & \text{if } \forall j \in N \; \dot{\mathbf{p}}(j) = 0. \end{cases}$$

Theorem 2. *The unforced dynamic system $\ddot{y} = -F(\dot{y})$ with friction F described by (8)–(10) is globally asymptotically stable if and only if at least one elasto-plastic element j is slipping in terms of $\dot{x}_j = \dot{y}$.*

Corollary 2. *The dynamic system in terms of Theorem 2 is globally asymptotically stable in Lyapunov sense if and only if at least one elasto-plastic element j has $k_j \to \infty$.*

It appears the initial stiffness at motion reversals is crucial for pre-sliding damping

$$\left.\frac{\delta F}{\delta x} \to \infty \right|_{\text{upon reversals}}$$



T. Koizumi and H. Shibazaki, "A study of the relationships governing starting rolling friction," *Wear*, vol. 93, no. 3, pp. 281–290, 1984.

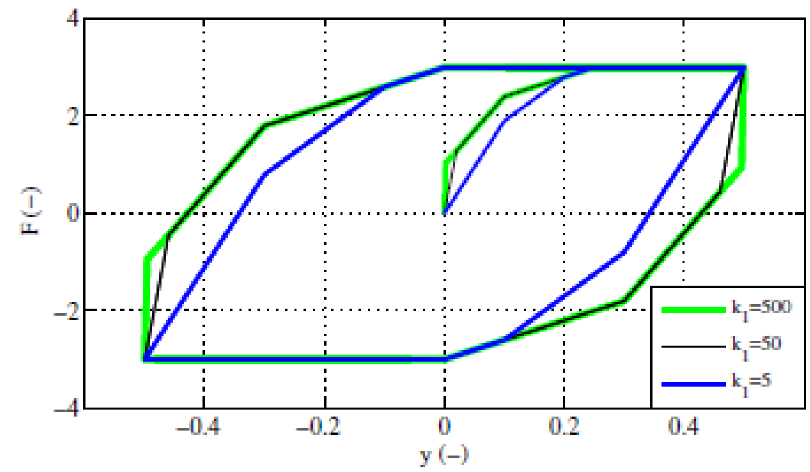
Nonlinear damping by PI operator, cont.

- Initial contact stiffness crucial for damping at motion reversals

Infinite initial stiffness at motion reversals

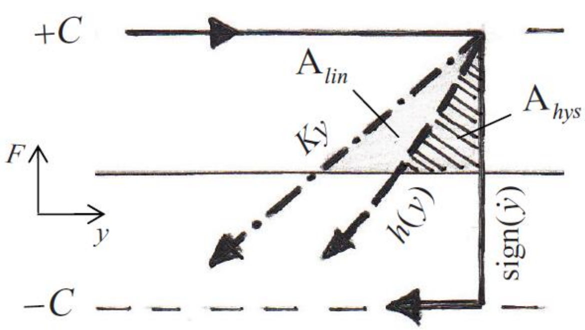
$$\frac{\partial F(t_r^+)}{\partial y(t_r^+)}\bigg|_{y=y(t_r)} = \sum_{j=1}^{N} k_j$$

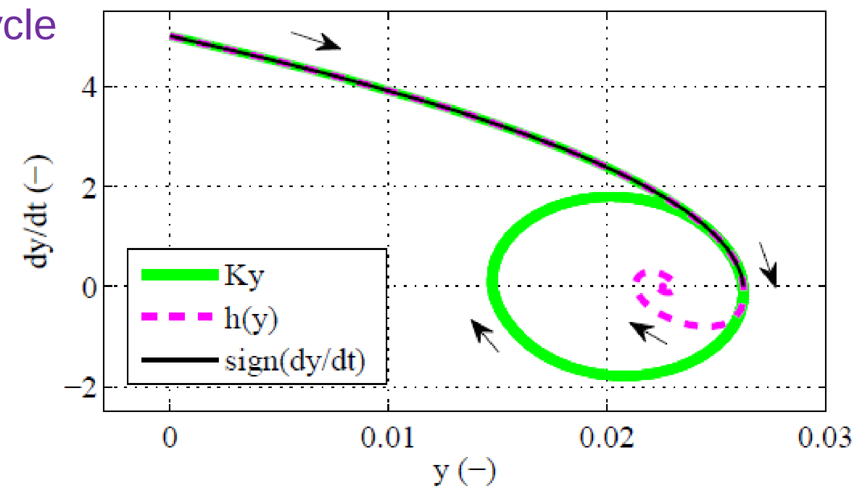
$$F(y,\dot{y}) = c_1\,\mathrm{sign}(\dot{y}) + \sum_{j=2}^{N} F_j$$



Convergence after reversal

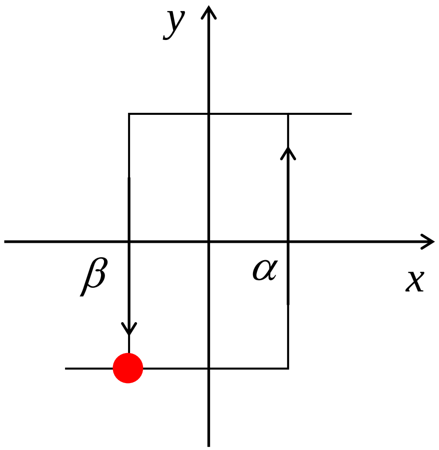
- constant damping rate (bound. case) → Coulomb
- zero damping rate (bound. case) → limit cycle
- varying (hysteresis) damping rate





Dissipation by counterclockwise hysteresis

- Preisach operator of hysteresis relay



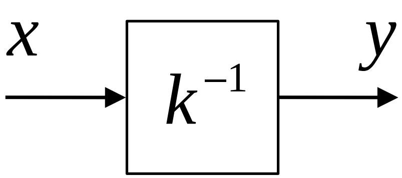
$$y(t) = \min[\mathrm{sign}(x-\beta), \max[y(t_-), \mathrm{sign}(x-\alpha)]]$$

$$y(t_0) = \begin{cases} \mathrm{sign}(x(t_0)), & \text{if } x(t_0) \in (-\infty, \beta) \vee (\alpha, \infty) \\ [-1, +1], & \text{otherwise.} \end{cases}$$

M Ruderman. Computationally efficient formulation of relay operator for Preisach hysteresis modeling. IEEE Transactions on Magnetics, 51 (12), 1-4, 2015

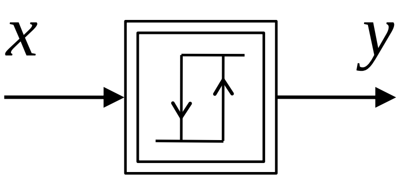
- Linear versus Preisach hysteresis transducer

Linear gain (spring, Hooke's law)



$$y(t) = k^{-1} x(t)$$

Counterclockwise (CCW) hysteresis

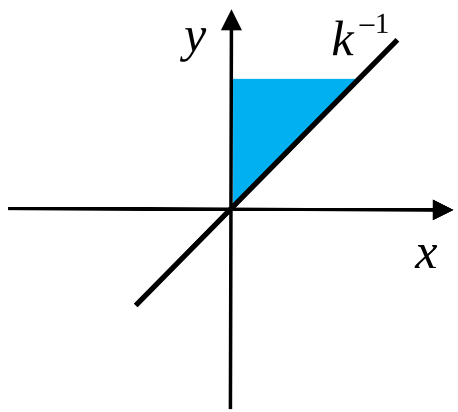


$$y(t) = P\left[t, y_0\right] x(t)$$

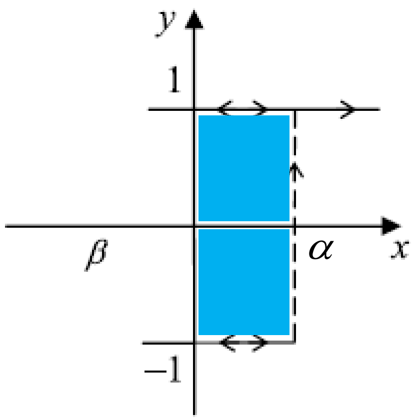
Dissipation by CCW hysteresis, cont.

- Energy at input cycles

Linear gain (spring, Hooke's law)



Counterclockwise (CCW) hysteresis



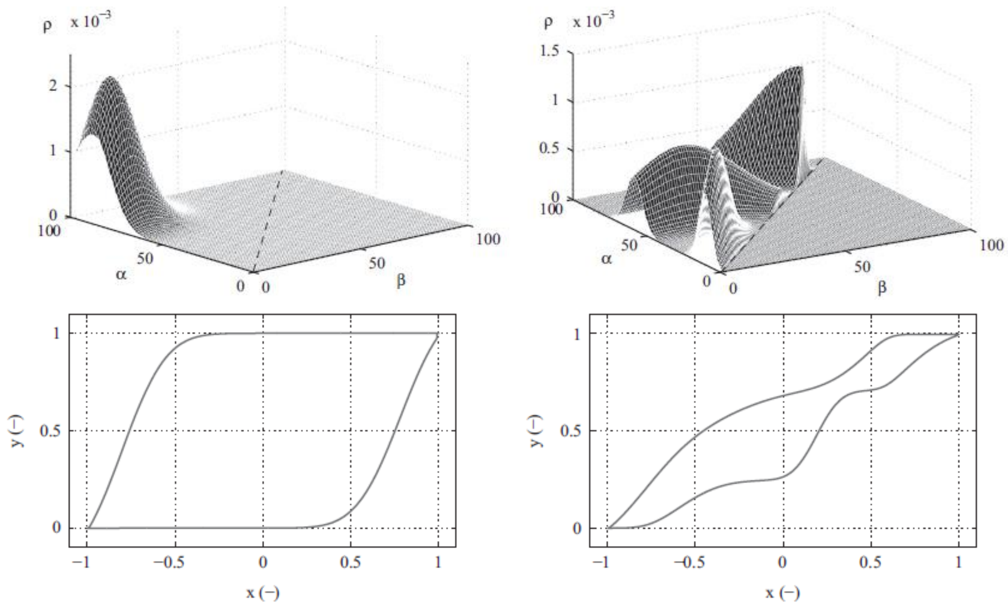
Considering power of the input-output pair and then integrating over time

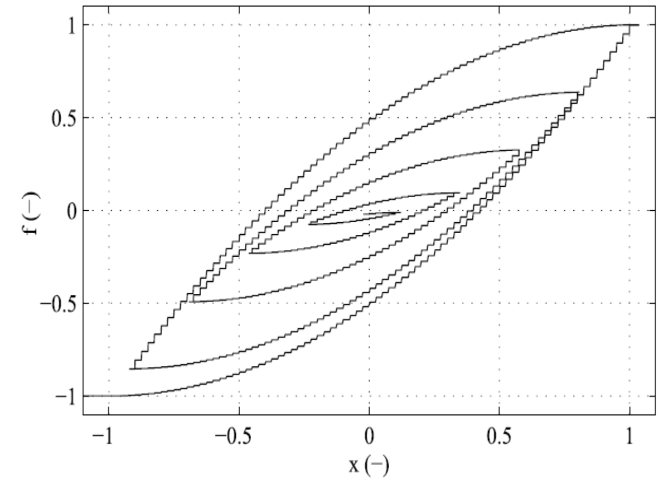
$$E = \int W dt = \int k y \dot{y} dt =$$
$$= k \int y dy = \frac{1}{2} k y^2$$

$$E = \int W dt = \int x \dot{y} dt =$$
$$= \int_0^{T^-} x 0 dt + \int_{T^-}^{T^+} \alpha \dot{y} dt + \int_{T^+}^{t} x 0 dt =$$
$$= \alpha \int_{T^-}^{T^+} 2\delta dt = 2\alpha$$

Dissipation by CCW hysteresis, cont.

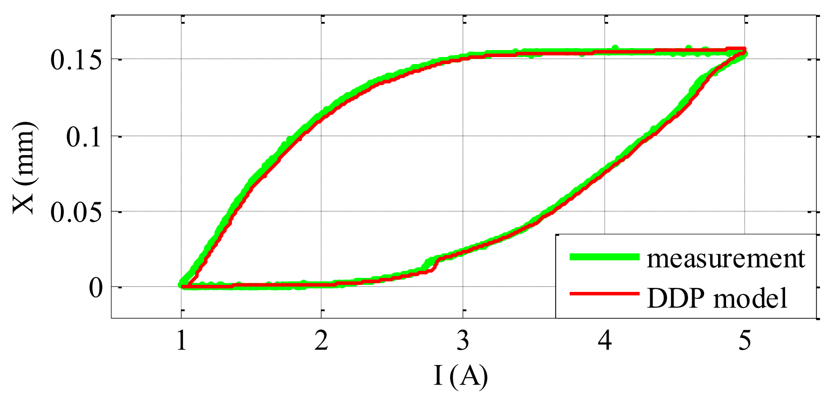
- Preisach operator as feedforward rate-independent damping



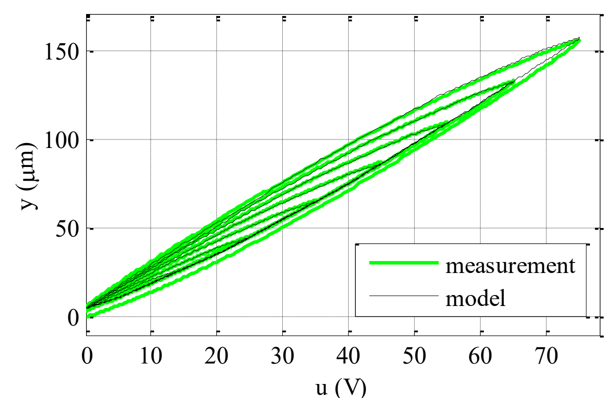


- Transducer examples:

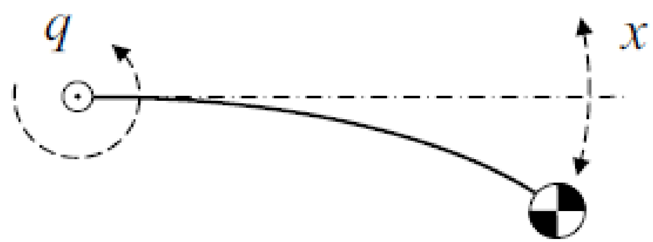
MSM



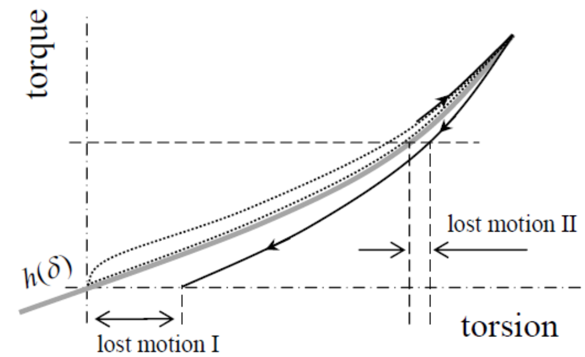
Piezoelectric

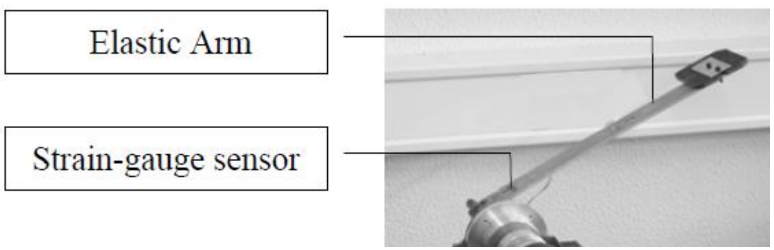


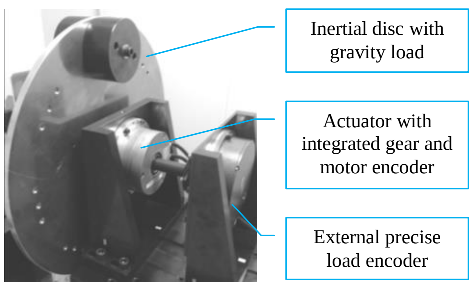
- Nonlinear damping in feedback path of system dynamics

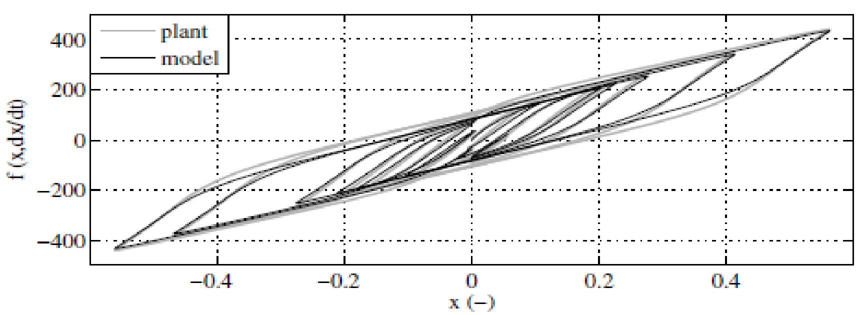


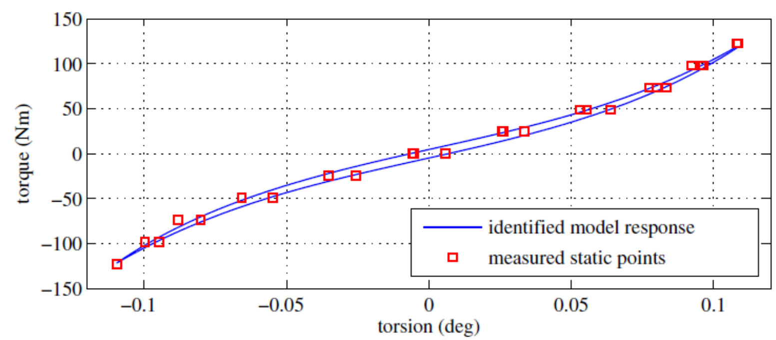
$$a\,\ddot{x} + f(x,\dot{x}) = b\,\ddot{q}$$







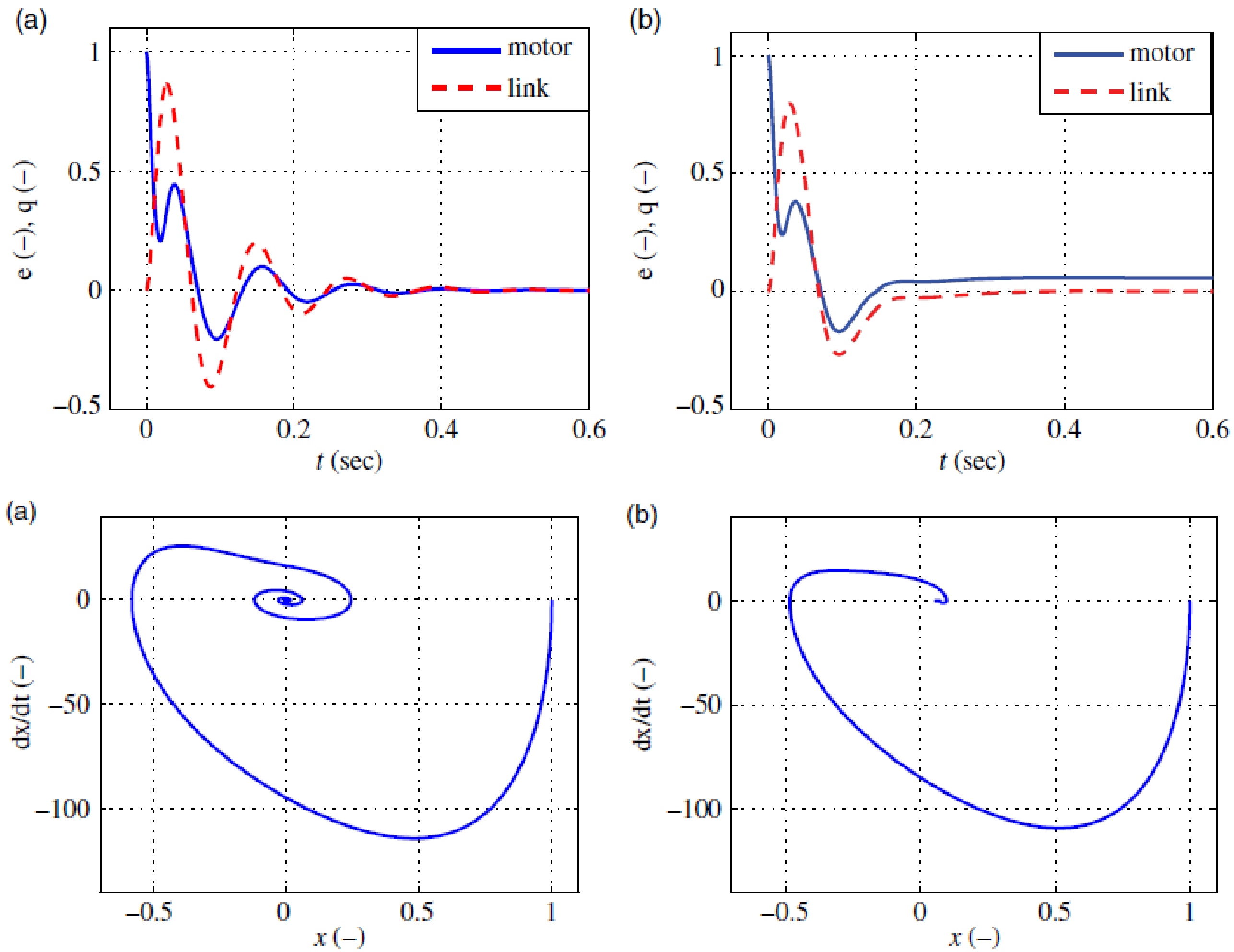




Dissipation by CW hysteresis, cont.

- Auxiliary nonlinear (structural) damping in elastic joints

Linear joint (4th order system) | Hysteretic joint (4th order system)



Dissipation by CW hysteresis, cont.

Considering a static hysteresis map with the input y and output ξ, which represent "energy pair" of a physical system, one can introduce the supply rate $w = \dot{y}(t) \cdot \xi(t)$. The energy flowing into the system, I/O hysteresis in our case, is obtained as

$$\int_{t_1}^{t_2} w \mathrm{d}t.$$

Then, for a non-negative real function $V(z) : Z \to \mathbb{R}^+$ of the system state z, called the storage function, the system is said to be *dissipative*, cf. [37], when satisfying

$$V(z(t_2)) - V(z(t_1)) \leq \int_{t_1}^{t_2} w \mathrm{d}t \tag{1}$$

Changes in energy level between two operational points

$$E(y_2) - E(y_1) = \int_{y_1}^{y_2} \xi(y) \mathrm{d}y$$

If there is no hysteresis in $\xi(y)$

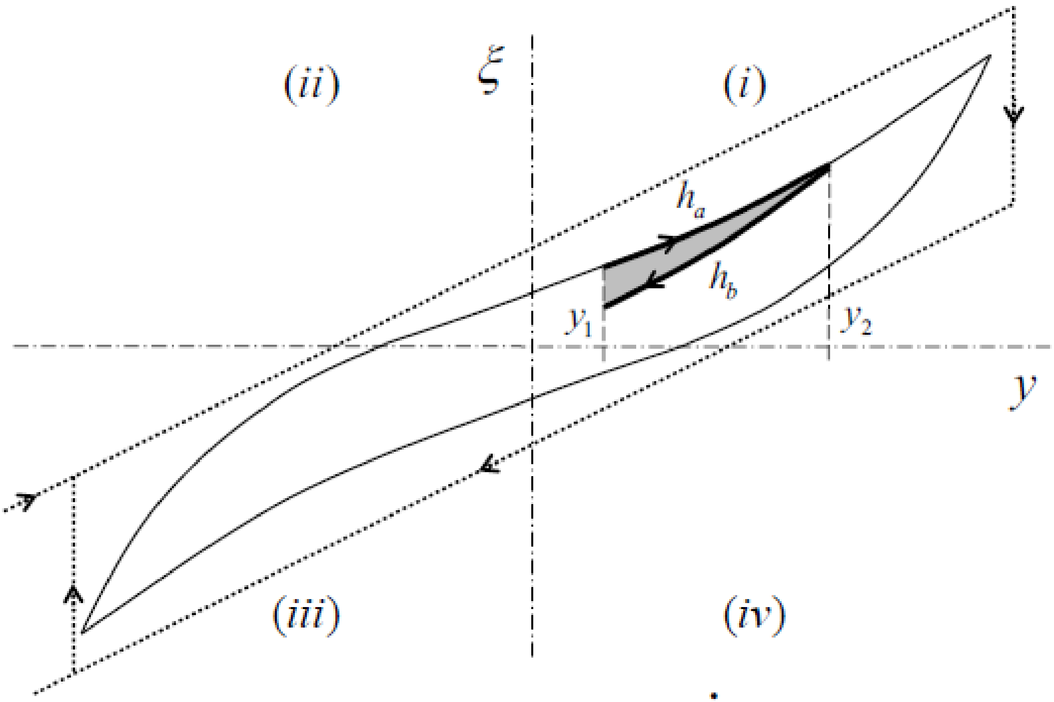
$$\begin{aligned} \Delta E_{2-1} &= \int_{y_1}^{y_2} \xi_a(y) \mathrm{d}y = - \int_{y_2}^{y_1} \xi_b(y) \mathrm{d}y, \\ \Delta E_{1-2} &= \int_{y_2}^{y_1} \xi_b(y) \mathrm{d}y = - \int_{y_1}^{y_2} \xi_a(y) \mathrm{d}y. \end{aligned}$$

total energy change during one y_1-y_2-y_1 cycle is $\Delta E_{2-1} + \Delta E_{1-2} = 0$

Dissipation by CW hysteresis, cont.

- All hysteresis maps (non-zero area upon reversal) are dissipative

$$\Delta E = \Delta E_{2-1} + \Delta E_{1-2} = \int_{y_1}^{y_2} \xi_a(y)\mathrm{d}y - \int_{y_2}^{y_1} \xi_b(y)\mathrm{d}y \neq 0$$



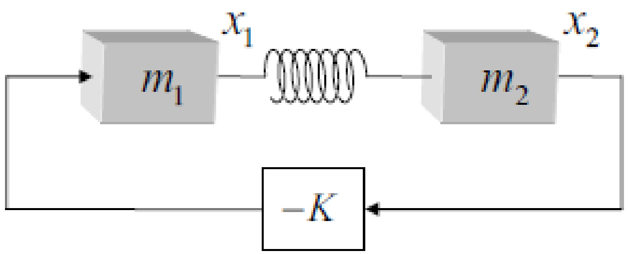
$$\xi(y) = g(y) + h\,\mathrm{sign}(\dot{y})$$

$$V = \frac{1}{2}\gamma\, y^2 - \int_Y \xi(y)\mathrm{d}y$$

$$\dot{V} = \gamma\, y\, \dot{y} - \gamma\, y\, \dot{y} - h\,\mathrm{sign}(\dot{y})\,\dot{y} = -h|\dot{y}|$$

Dissipation by CW hysteresis, cont.

- Locally stabilizing behavior of CW hysteresis in the loop



$$\Phi = \begin{cases} \dot{x} = Ax + Bu, \\ y = C^T x, \\ -u = \xi[y, t, \xi_0] \end{cases}$$

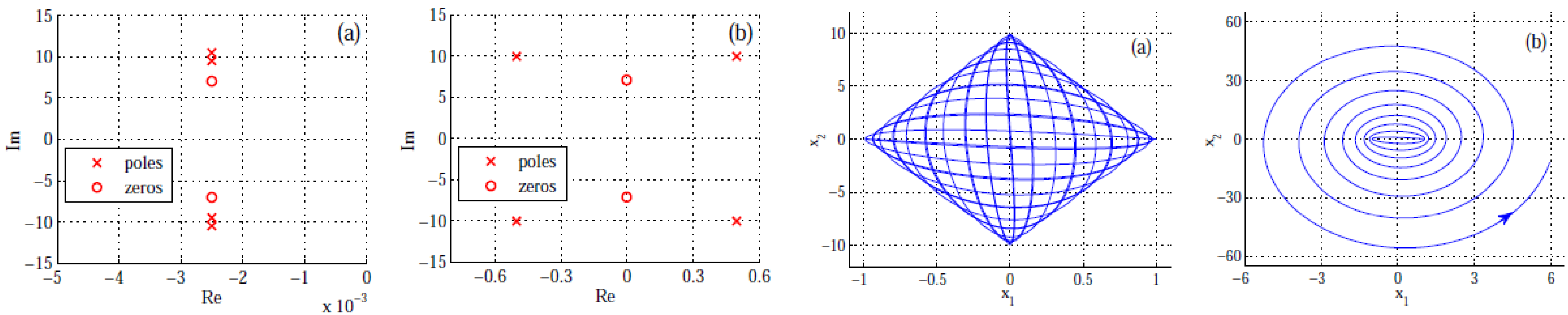
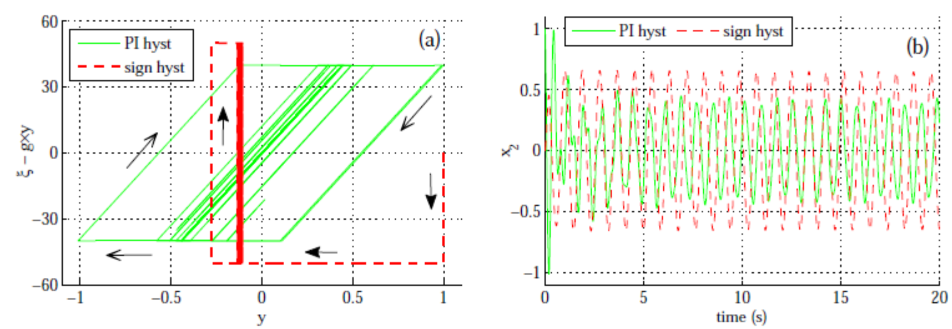


FIGURE 7. Poles and zeros for boundary-stable $K = 99$ (a) and boundary-unstable $K = 101$ (b) feedback

When adding hysteresis to the linear spring, local stabilization (depending on initial conditions) implies

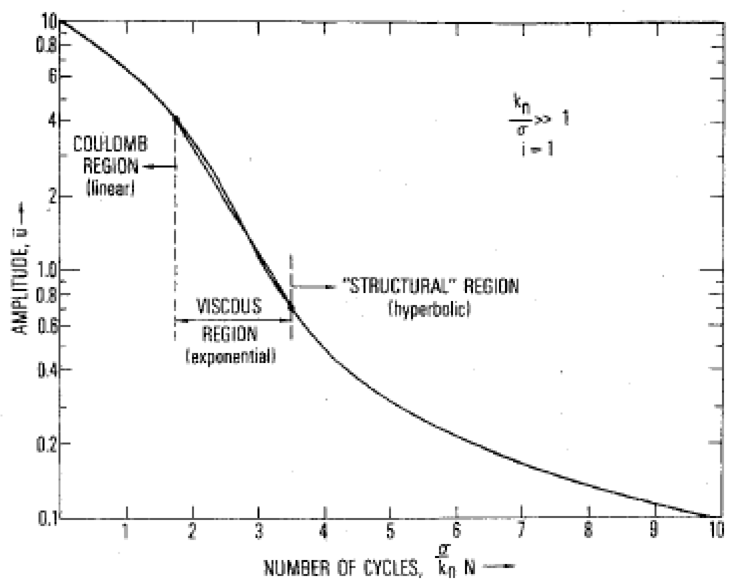


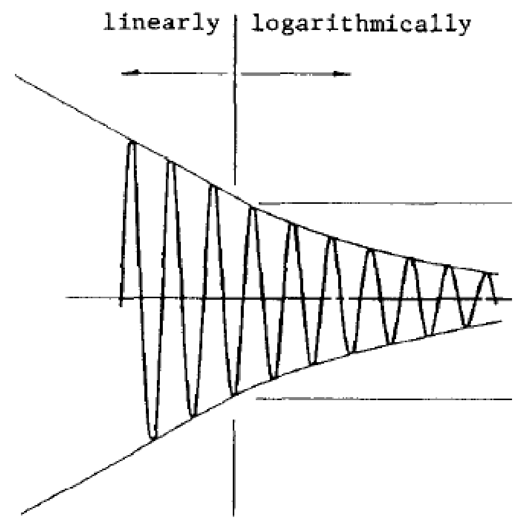
- Energy losses in motion systems

$$m\ddot{x} + b\dot{x} + kx = 0 \qquad\qquad m\ddot{x} + b\,\mathrm{sign}(\dot{x}) + kx = 0$$

$$\dot{E} = \dot{x}(kx + m\ddot{x}) = -b\dot{x}^2 \qquad\qquad \dot{E} = -b|\dot{x}|$$

- Time- and/or state-varying system damping





- Damping ratios: exponential; constant; something else…

Thank you for attention

Acknowledgement:
partially supported by

This research receives funding from the European Community Horizon 2020 Research and Innovation Programme (H2020-MSCA-RISE-2016) under the Marie Skłodowska-Curie grant agreement No 734832

Nonlinear damping in system dynamics and control

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Abstract

Damping is a natural way to extract energy from an autonomous, to say isolated, dynamic system. Physically it means an irreversible dissipation process, mostly through heat transfer, while in the control systems it is inherently associated with stabilization of the state solutions, correspondingly convergence to a stable equilibrium. Nonlinear damping is relevant but, at the same time, not trivial for both – analysis and modeling of the dynamic systems and controller synthesis for their efficient regulation. This talk, devoted to nonlinear damping, will consist of two parts. In the first one, we will analyze some nonlinear damping phenomena, while focusing on rate-independent (sometimes called structural or hysteresis) damping and discussing dissipation power, local stabilizing properties, and invariant sets of equilibria. The second part of the talk will discuss nonlinear damping in the feedback control, with focus on the phase-plane analysis of solution trajectories. A novel nonlinear damping control with optimal convergence without transient overshoot will also be illustrated in detail.

Bio

Michael Ruderman received the Dr.-Ing. in electrical engineering from the Technical University (TU) Dortmund, Dortmund, Germany, in 2012. During 2006–2013, he was a Research Associate with the Institute of Control Theory and Systems Engineering, TU Dortmund. In 2013–2015, he was with Nagoya Institute of Technology, Nagoya, Japan, as specially appointed Assistant Professor. In 2015 he was specially appointed Associate Professor with the Department of Electrical Engineering, Nagaoka University of Technology, Nagaoka, Japan, before joining in the same year the University of Agder (UiA), Grimstad, Norway. Since 2020 he is a full professor at UiA, teaching control theory in Master and PhD programs. His current research interests are in the motion control, robotics, nonlinear systems with memory, and hybrid control systems. He also serves in different editorial boards and technical committees of IEEE and IFAC societies and is chairing IEEE/IES TC on Motion Control.