UNIVERSITETET I AGDER

Robustly estimating and compensating uncertain oscillatory output

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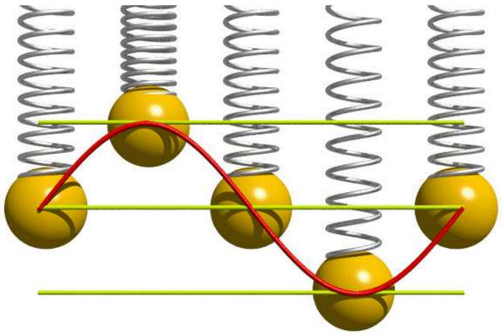
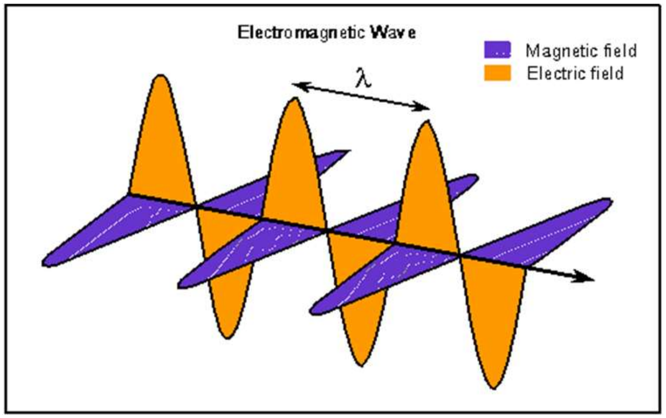
25.04.2024

Guest seminar

Loughborough University

- Robust frequency estimation





Measurement, analysis, monitoring of oscillatory quantities in almost all types of the systems:

mechanical vibrations, electric & power signals, electromagnetic waves, bio-medical signals

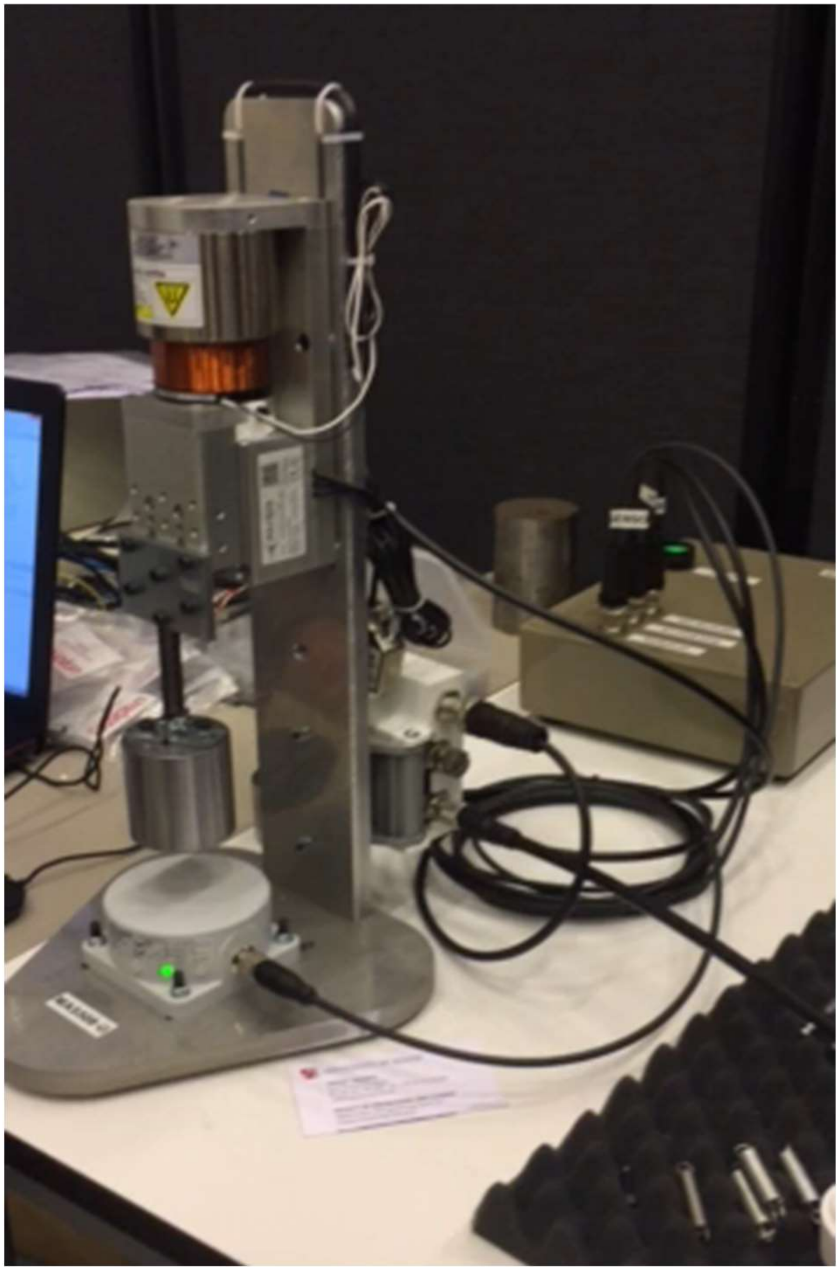
General issues with oscillatory output signals: bias, noise, time-varying parameters

$$y(t) = Y_0 + Y(t)\sin\big(\omega(t)\cdot t + \varphi\big) + \eta(t)$$

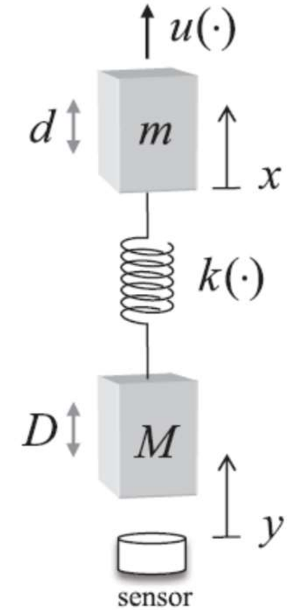
ref. [5]

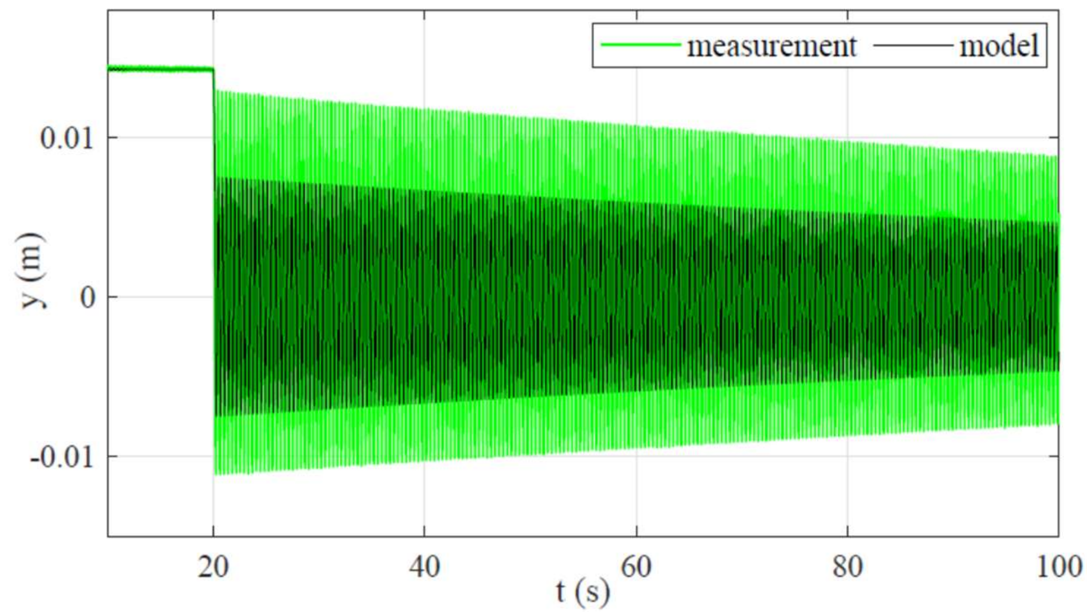
Numerous approaches from last three decades (including various PLLs), also the global estimator

Experimental system in use



Two-mass actuator-load system (anno 2020) for oscillations analysis & control prototyping





Robust frequency estimation

Robust estimator: background

Mechanical Systems and Signal Processing 170 (2022) 108756

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ELSEVIER

One-parameter robust global frequency estimator for slowly varying amplitude and noisy oscillations

Michael Ruderman

University of Agder, 4604, Norway

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 44, NO. 4, APRIL 1999

A Globally Convergent Frequency Estimator

Liu Hsu, *Member, IEEE*, Romeo Ortega, *Fellow, IEEE*, and Gilney Damm, *Associate Member, IEEE*

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 49, NO. 2, FEBRUARY 2004

An Adaptive Notch Filter for Frequency Estimation of a Periodic Signal

M. Mojiri and A. R. Bakhshai

- One-parameter robust frequency estimator [1], motivated by [5], for noisy harmonic signals with a slowly-varying amplitude

$$\sigma(t) = k(t)\sin\big(\omega_0 t\big) + \eta(t) \qquad \text{eq. (1)}$$

✓ Amplitude variations are slow (comparing to ω_0), and k appears as a 'frozen' term in analysis

✓ Measurement noise is zero-mean with a constant power spectral density (PSD), i.e. white-noise

$$PSD\{\eta(\omega)\} = \text{const} \equiv p$$

- Original global frequency estimator [4], [5] (adaptive notch filter)

$$\ddot{x}(t) + 2\zeta\theta\dot{x}(t) + \theta^2 x(t) = \theta^2\sigma(t), \qquad \dot{\theta} = -\gamma x(t)\big(\theta^2\sigma(t) - 2\zeta\theta\dot{x}(t)\big)$$

Robust estimator: structure and I/O properties

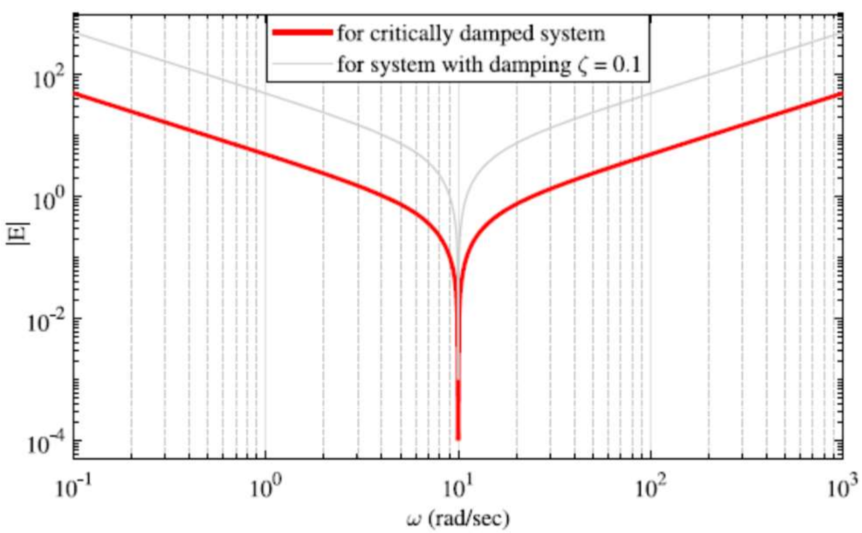
- One-parameter robust frequency estimator [1], eqs. (4)-(6)

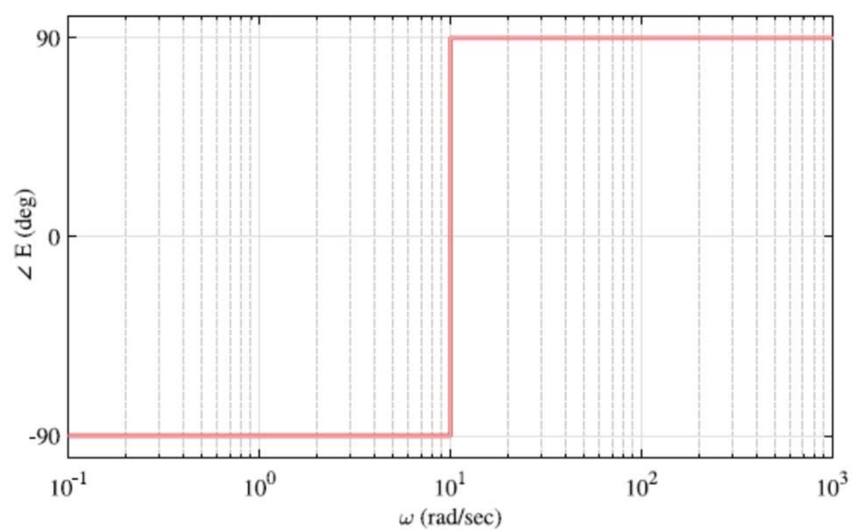
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\theta^2 & -2\theta\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 2\theta\end{bmatrix}\sigma, \quad y = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}, \quad \dot{\theta} = -\gamma\,\mathrm{sign}(x_1)(\sigma - y)$$

at the angular frequency $\theta = \omega_0 \quad \Rightarrow \quad e = \sigma - y \to 0$

$$G(j\omega) = \frac{y(j\omega)}{\sigma(j\omega)} = C^T(j\omega I - A)^{-1}B \quad \Rightarrow \quad e(j\omega) = (1 - G)G^{-1}y(j\omega)$$

$$E(j\omega) = \frac{e(j\omega)}{y(j\omega)} = \big(1 - G(j\omega)\big)G(j\omega)^{-1}$$





Robust estimator: asymptotic convergence

Theorem 1. *The frequency estimator (4)–(6) is global for (1) and converges asymptotically as $\theta(t) \to \omega_0$ for $t \to \infty$, regardless of the $\theta(0) > 0$ initialization, provided the small adaptation gains $\gamma > 0$ and slowly varying amplitudes $k(t)$. The frequency-estimation error $\varepsilon(t) = \omega_0 - \theta(t)$ converges uniformly and exponentially in terms of*

$$\left|\varepsilon(t_2)\right| < \alpha \left|\varepsilon(t_1)\right| \exp\left(-\beta(t_2 - t_1)\right) \tag{9}$$

for some $\alpha > 0$ and $\forall\, t_2 > t_1$. The exponential rate of convergence is independent of $\eta(t)$ noise and as follows:

$$\beta = 0.5\, \gamma k\, \omega_0^{-1} + \delta, \tag{10}$$

where δ is a small positive constant independent of γ, k, ω_0.

- Sketch of the proof (for more details see [1])

Let $0 < \theta(0) < \omega_0$ be an arbitrary initialization

$$y(t) = b \sin(\omega_0 t + c) \qquad \text{where } b = k\left|G(j\omega_0)\right|$$

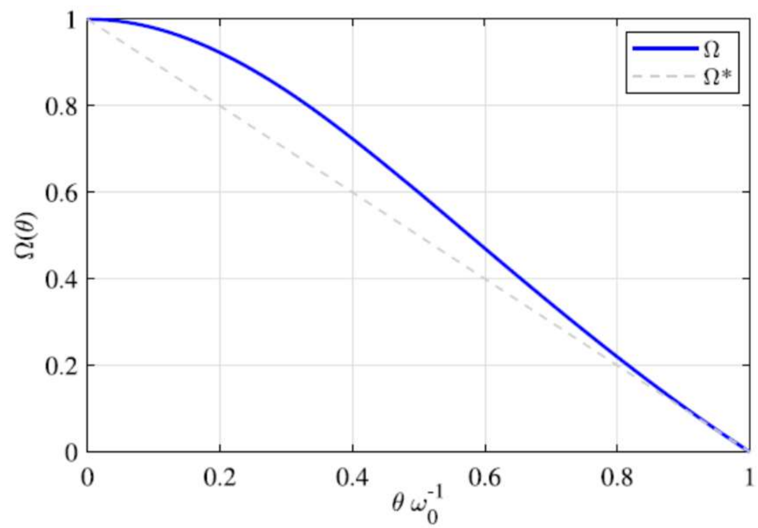
$$\Rightarrow \quad x_1(t) = -\frac{b}{\omega_0}\cos(\omega_0 t + c) = -\frac{b}{\omega_0}\sin(\omega_0 t + c + \pi/2)$$

$$\Rightarrow \quad e(t) = a \sin(\omega_0 t + c + \pi/2) \qquad \text{where } a = b|E(j\omega_0)|$$

$$\dot{\theta} = \gamma k \left|G(j\omega_0)\right| \left|\frac{1 - G(j\omega_0)}{G(j\omega_0)}\right| \left|\sin(\omega_0 t + c + \pi/2)\right|$$

It can be seen that for all $\gamma, k > 0$ the $\mathrm{sign}(\dot{\theta}) = +1$ as long as $\theta(t) < \omega_0$

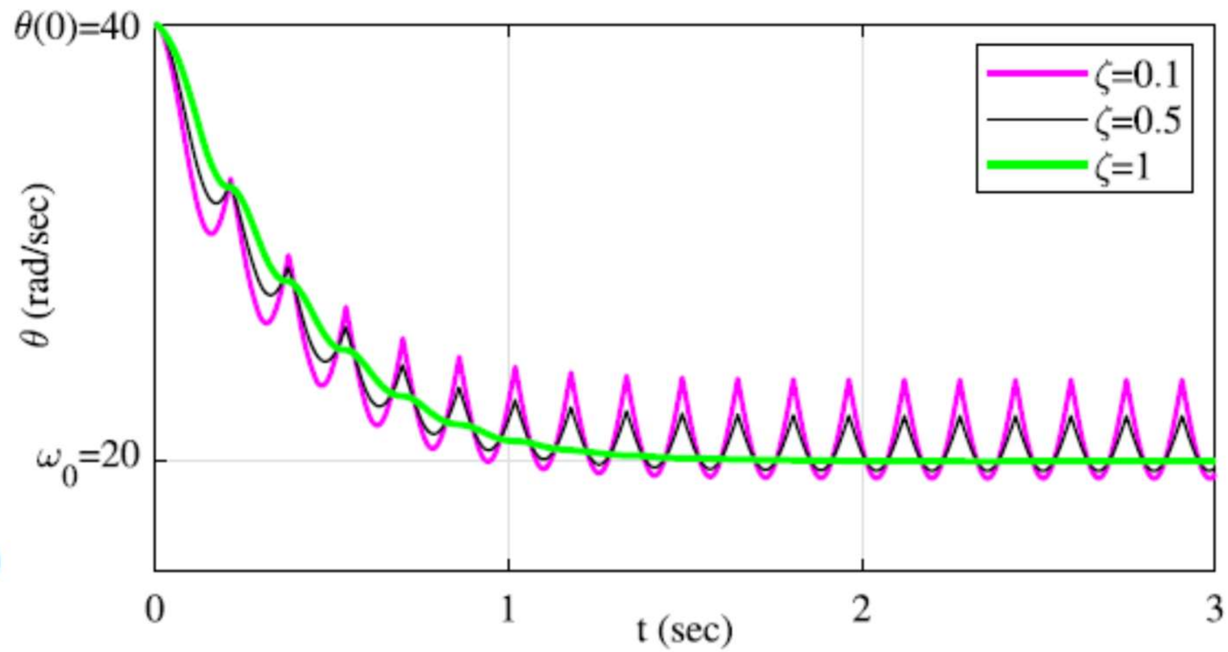
$$\left|G(j\omega_0)\right| \left|\frac{1 - G(j\omega_0)}{G(j\omega_0)}\right| = \frac{\left|\theta^2 - \omega_0^2\right|}{\theta^2 + \omega_0^2} \equiv \Omega(\theta)$$



$$\Omega^*(\theta) = -\theta\omega_0^{-1} + 1 \quad \Rightarrow \quad \dot{\theta}^* = 0.5\,\gamma k\left(-\theta^*\omega_0^{-1} + 1\right) \quad \Rightarrow \quad \dot{\theta}(t) + 0.5\gamma k\omega_0^{-1}\theta(t) = 0.5\gamma k\omega_0^{-1} \cdot \omega_0$$

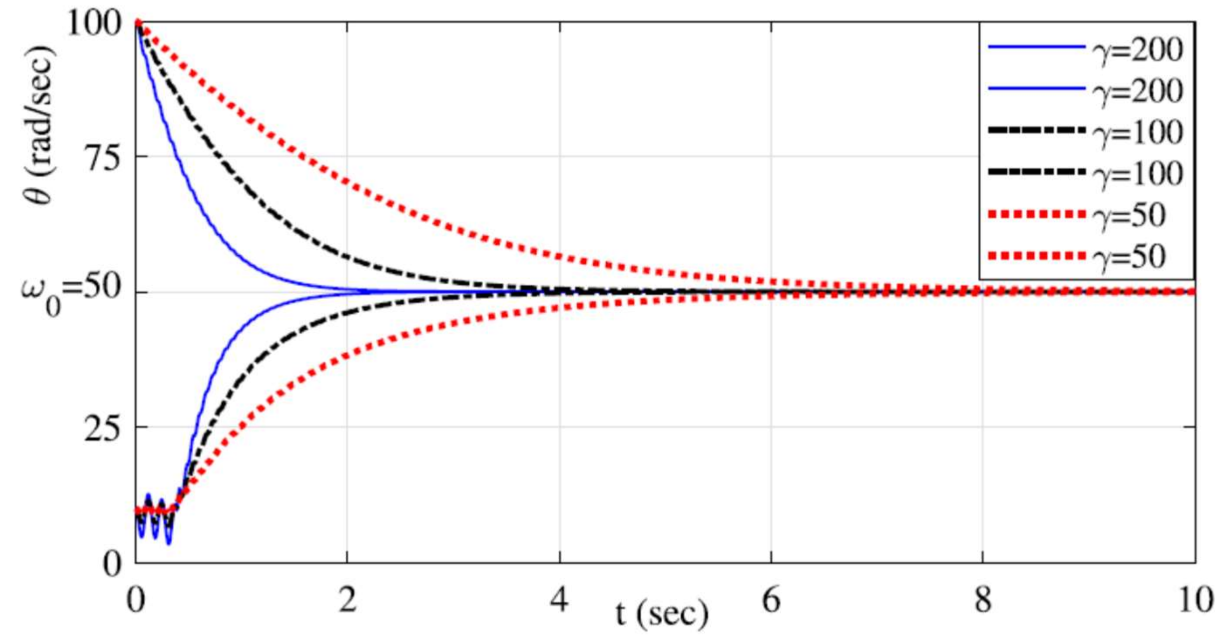
Robust estimator: convergence properties

- Optimality without damping parameter (i.e. for ζ=1)



estimator ($\gamma = 100$, $\omega_0 = 20$ rad/s)

- Convergence speed for various adaption gains γ



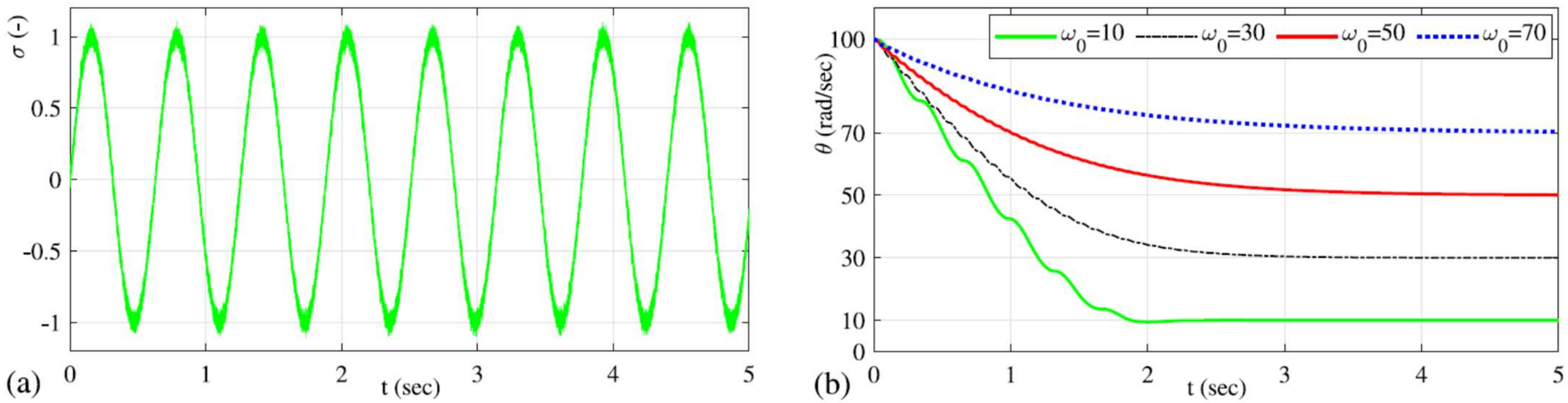
estimator

$\omega_0 = 50$ rad/s for $\sigma(t) = \sin(\omega_0 t)$

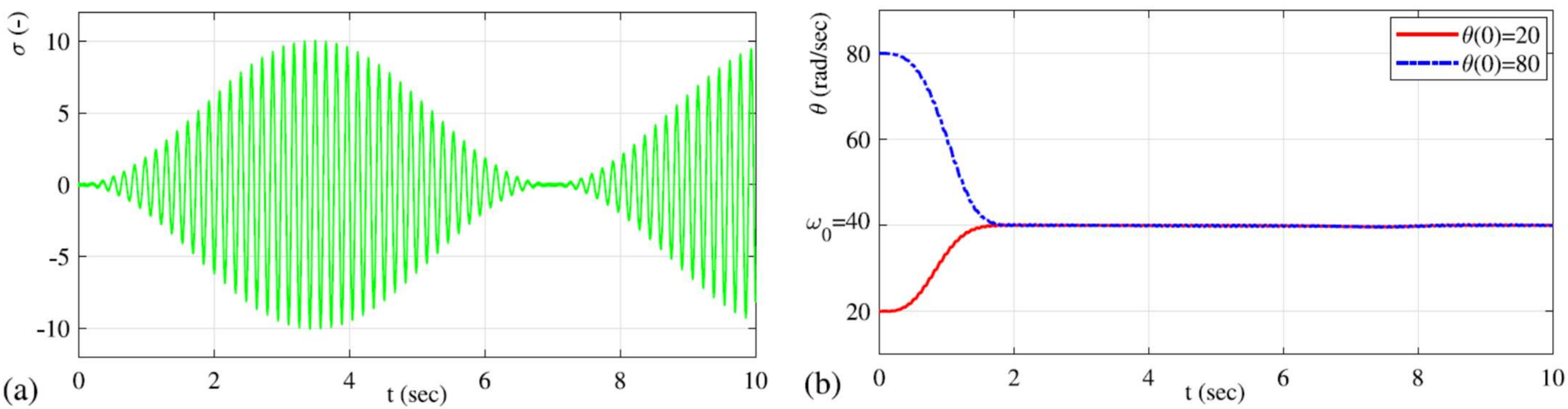
$\gamma = \{200, 100, 50\}$

Robust estimator: numerical examples

- Noisy signal, constant amplitude, various frequencies

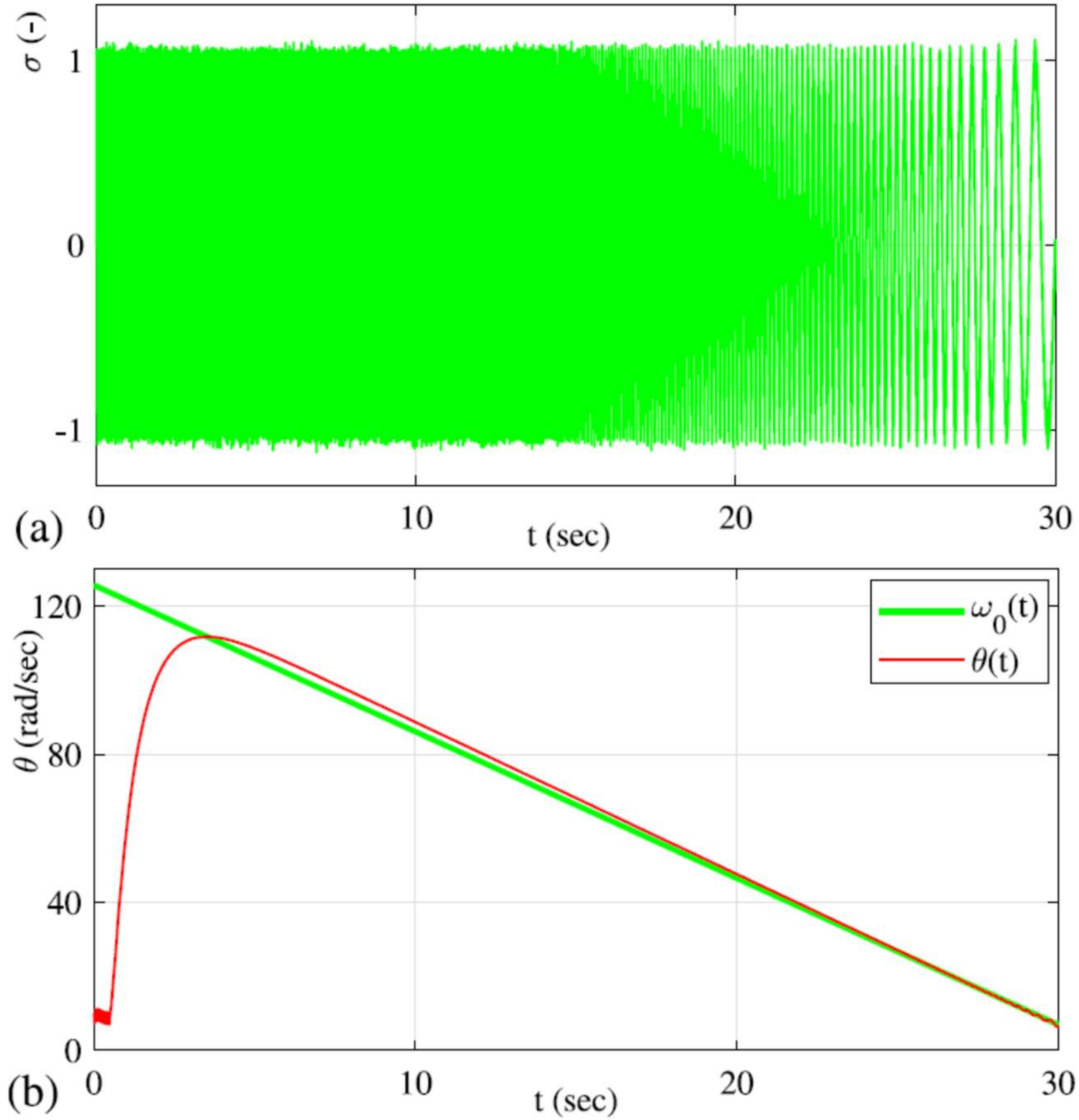


- Noisy signal, time-varying amplitude



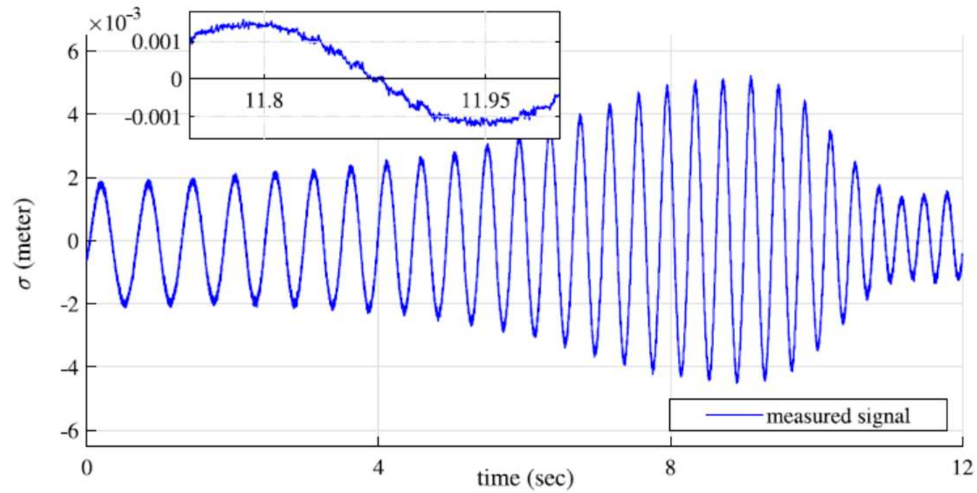
Robust estimator: numerical examples (cont.)

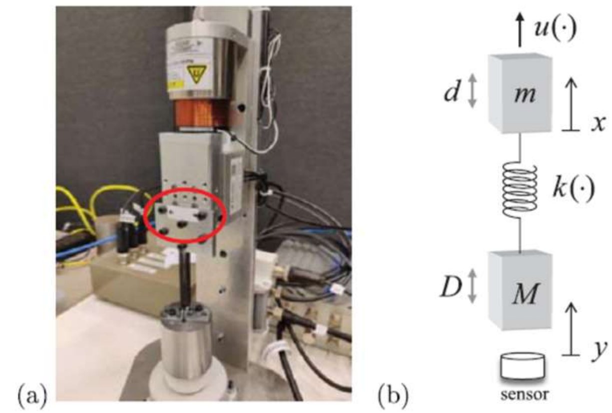
- Noisy signal, time-varying frequency (down-chirp signal)



Robust estimator: experimental benchmarking

- Noisy & biased signal with time-varying amplitude & frequency





- Comparison with the original global estimator [4]

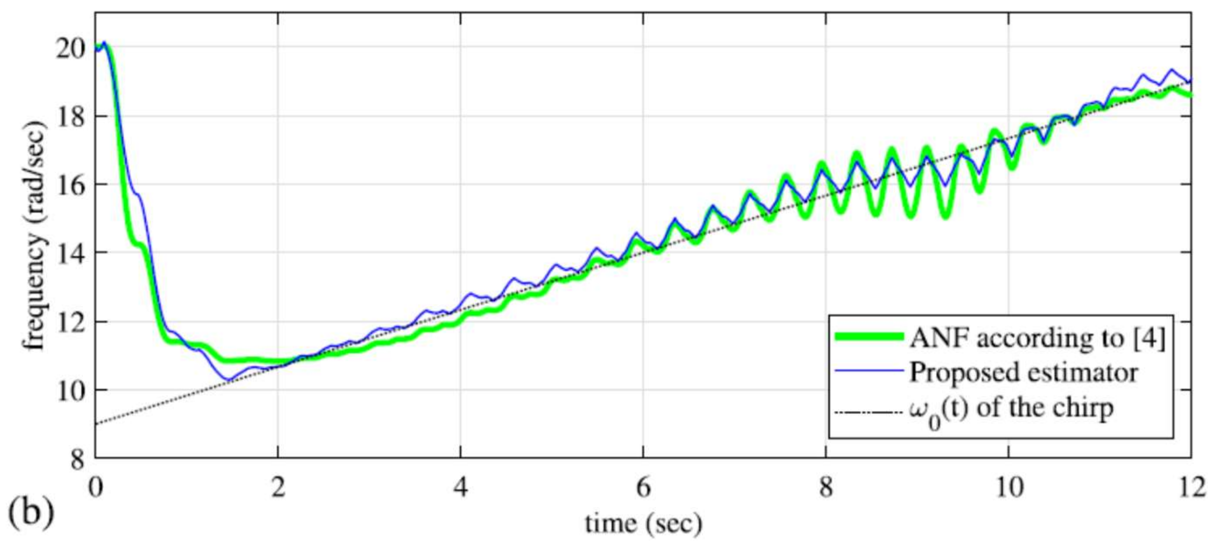
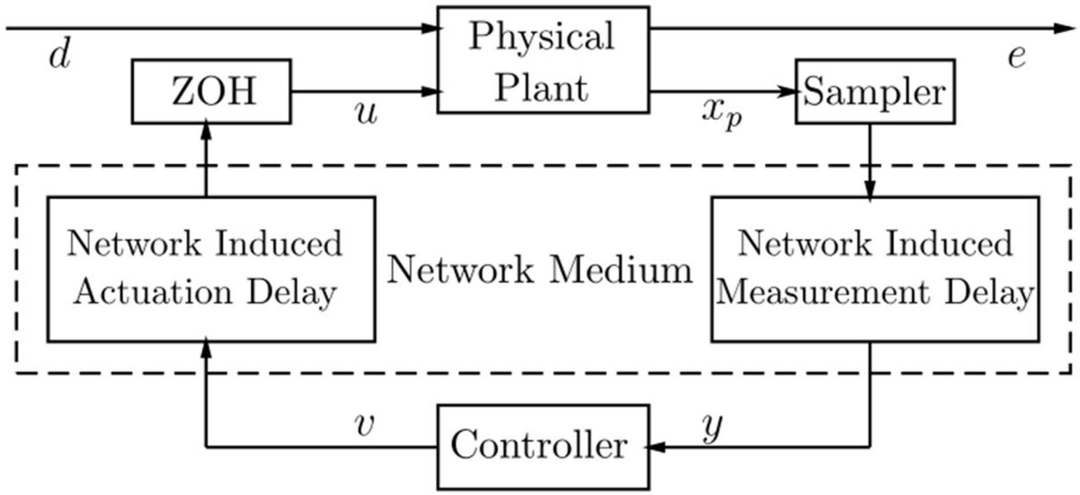


Table 1
Estimators' evaluation setting.

Set parameter	ANF according to [4]	Proposed estimator
$\theta(0)$ (rad/s)	20	20
γ	4e4	2e4
ζ	{0.7, 1, 1.3}	{0.7, 1, 1.3}

(Adaptive) time delay-based compensation

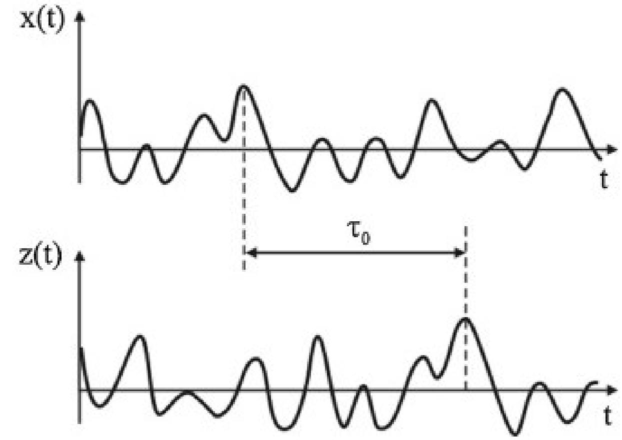
Motivation for using time delay



Shift of the time series by an (evtl. time-varying) factor

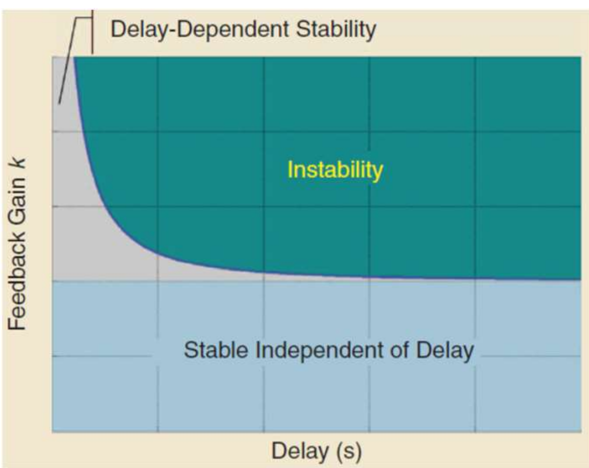
$$y(t) = x(t - \tau_0)$$

$$\tilde{G}(s) = G(s)\exp(-s\tau_0)$$



Usually, source of instability in feedback control systems

⇒ need for careful analysis and robust control design

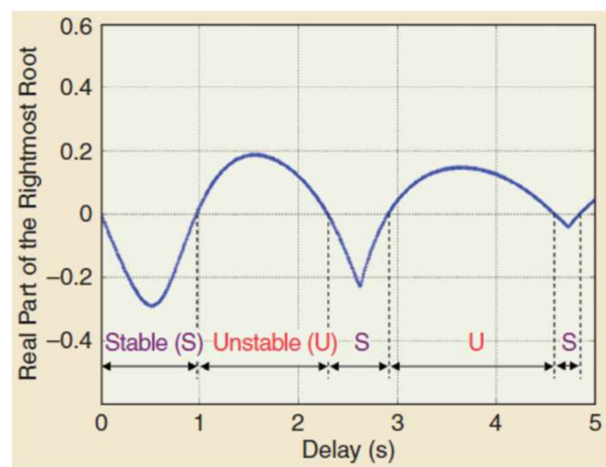


ref. [6]

$$\arg\left[\exp(-j\omega\tau_0)\right] = -\omega\tau_0$$

However, purposefully inserted time delays can be used for stabilization

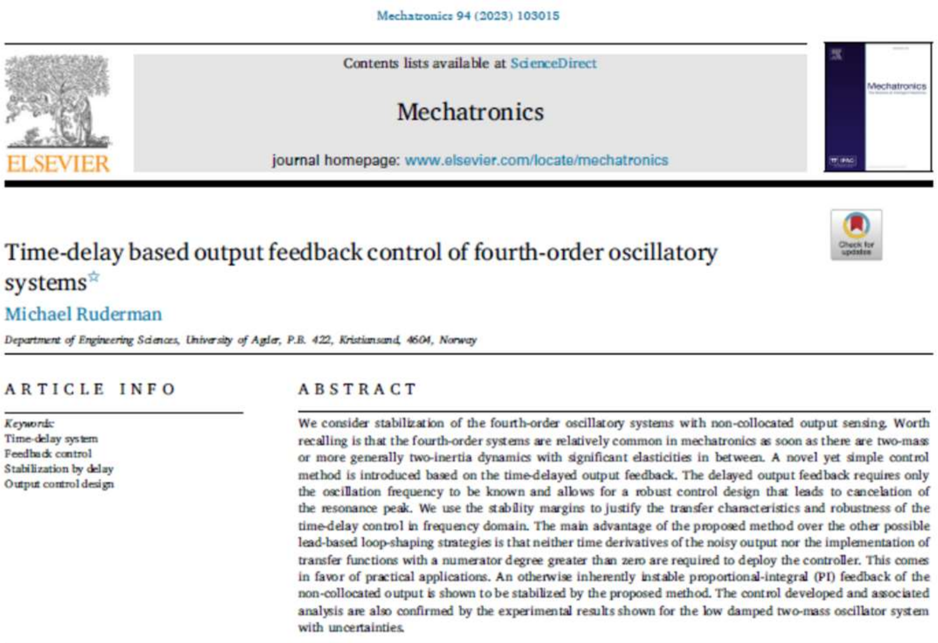
$$u = f\left(x(t), x(t-T)\right)$$



ref. [6]

Time delay-based output feedback compensator

- Original approach, initially anno 2021 [7], and elaborated in [2]



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Time-delay based output feedback control of fourth-order oscillatory systems☆

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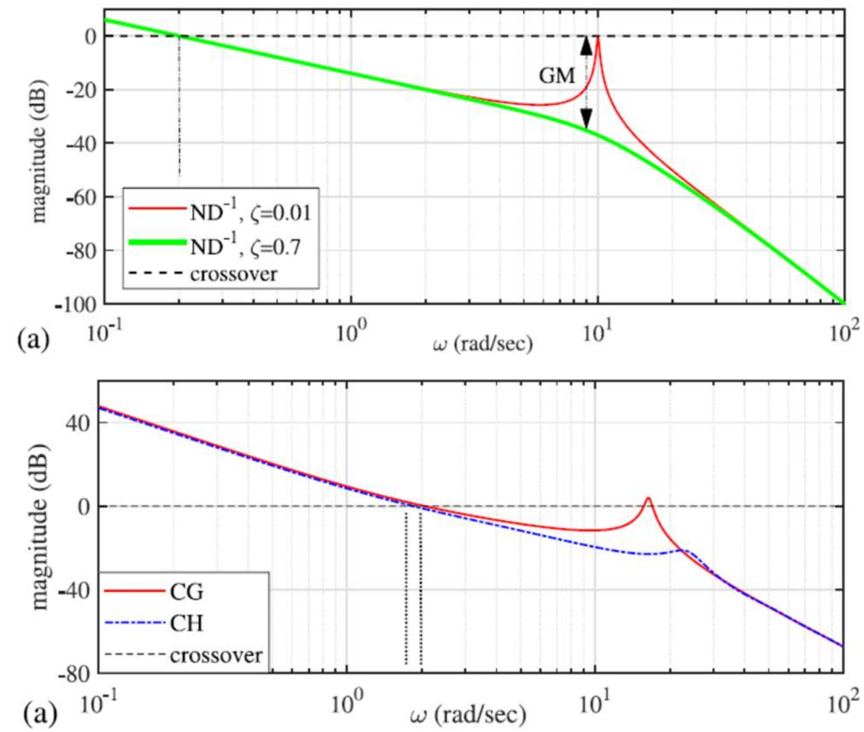
ARTICLE INFO

Keywords:
Time-delay system
Feedback control
Stabilization by delay
Output control design

ABSTRACT

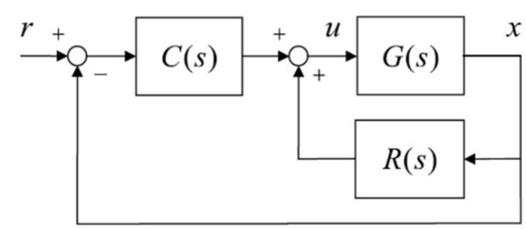
We consider stabilization of the fourth-order oscillatory systems with non-collocated output sensing. Worth recalling is that the fourth-order systems are relatively common in mechatronics as soon as there are two-mass or more generally two-inertia dynamics with significant elasticities in between. A novel yet simple control method is introduced based on the time-delayed output feedback. The delayed output feedback requires only the oscillation frequency to be known and allows for a robust control design that leads to cancelation of the resonance peak. We use the stability margins to justify the transfer characteristics and robustness of the time-delay control in frequency domain. The main advantage of the proposed method over the other possible lead-based loop-shaping strategies is that neither time derivatives of the noisy output nor the implementation of transfer functions with a numerator degree greater than zero are required to deploy the controller. This comes in favor of practical applications. An otherwise inherently instable proportional-integral (PI) feedback of the non-collocated output is shown to be stabilized by the proposed method. The control developed and associated analysis are also confirmed by the experimental results shown for the low damped two-mass oscillator system with uncertainties.



Research questions:

- ❑ Robust stabilization of non-collocated passive load
- ❑ Use of output feedback only ⇒ time-delay based control

Proposed (plug-in) time-delay based compensator (R) scheme

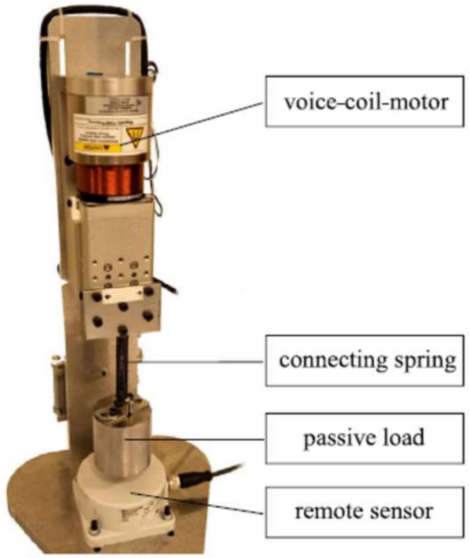


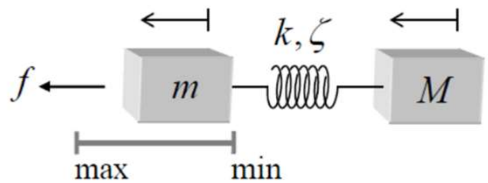
$$C(s)G(s) = K_p \frac{s + K_i K_p^{-1}}{s} \cdot \frac{N(s)}{D(s)} \quad \text{versus} \quad C(s)H(s) = \frac{C(s)N(s)}{D(s) + R(s)N(s)}$$

$$R(s) = K_d \left(\exp(-sT) - 1\right) \quad \text{with} \quad T = -\left[\arg G(j\omega_0)\right]\omega_0^{-1}$$

Experimental evaluation

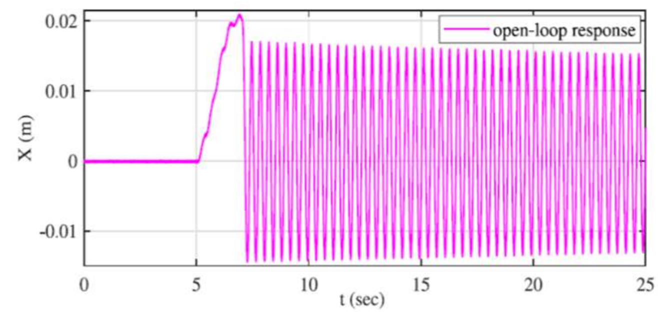
- Low-damped oscillatory non-collocated passive load



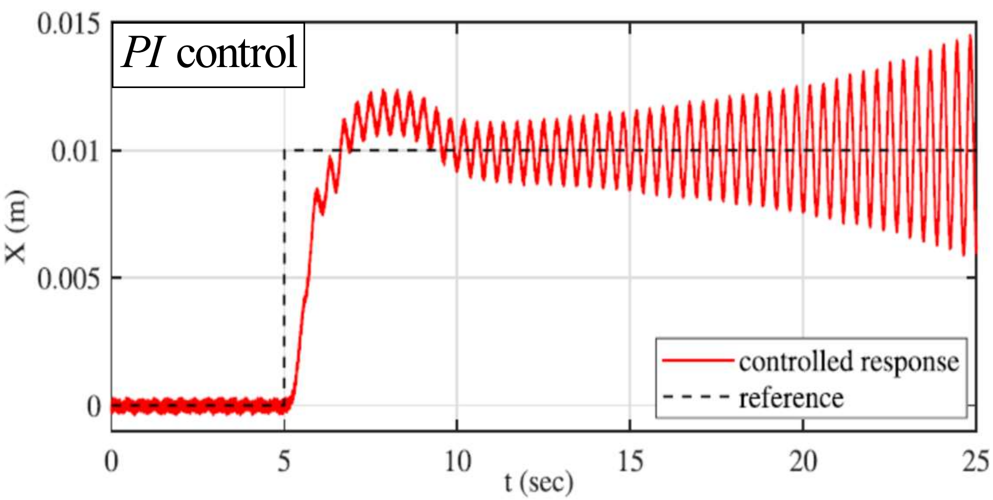


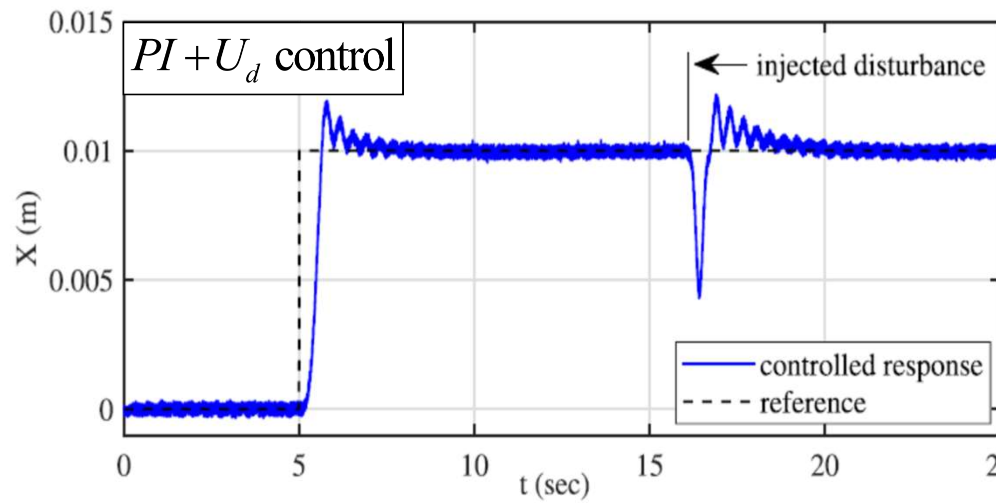
Nominal values of the system parameters.

Parameter	Unit	Value	Meaning
m_1	kg	0.6	Actuator mass
m_2	kg	0.75	Load mass
k	N/m	200	Spring constant
σ	kg/s	200	Actuator damping
δ	kg/s	0.01	Spring damping
R	V/A	5.23	Coil resistance
Ψ	V s/m	17.16	EMF constant
g	m/s^2	9.81	Gravity constant



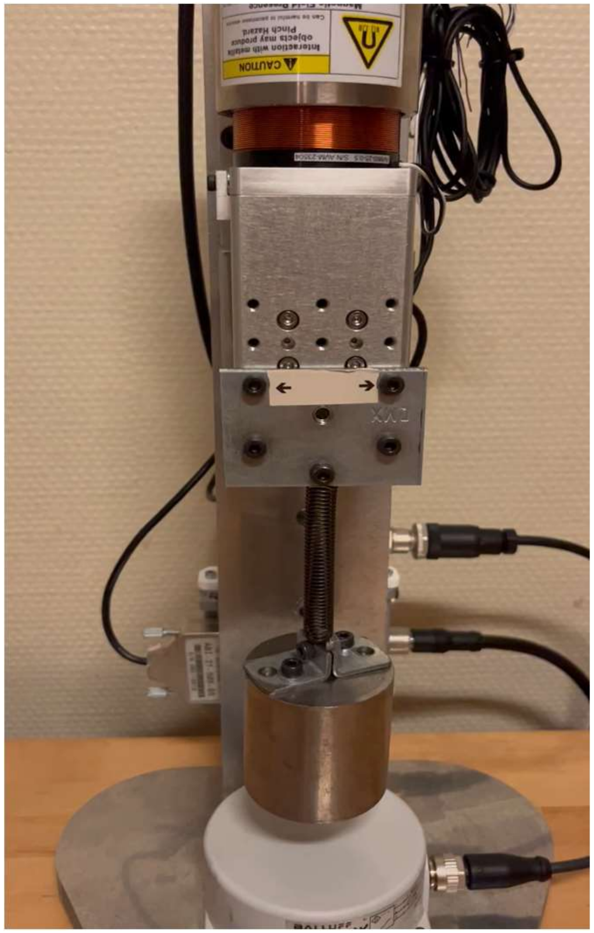
- PI feedback control versus PI + time-delay based compensator (U_d)



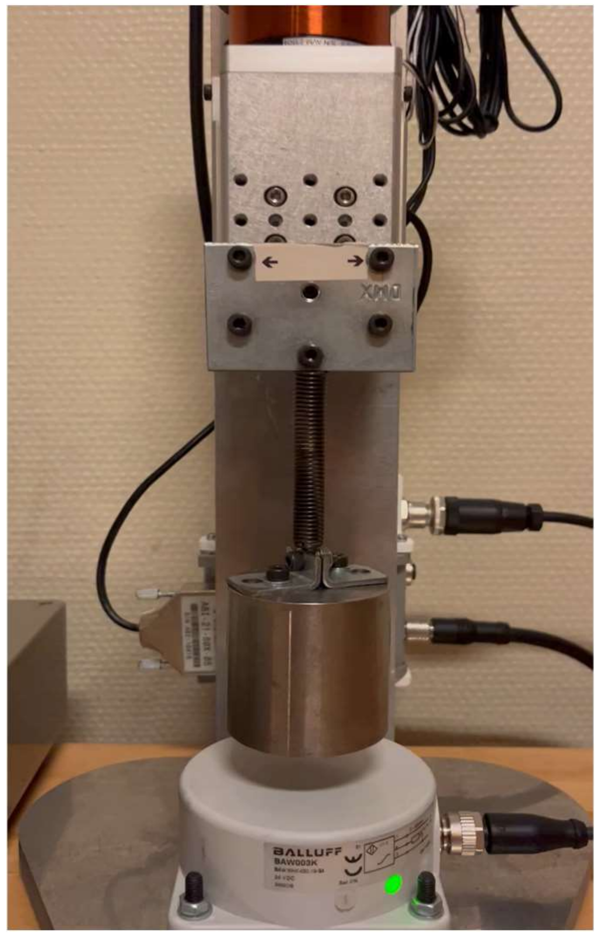


Experimental evaluation (movies)

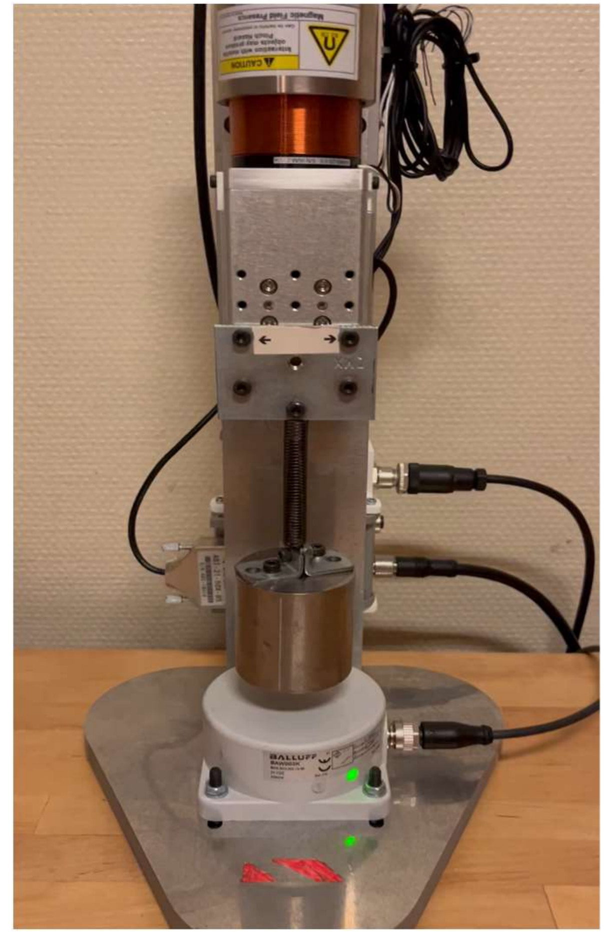
- Free-fall (uncontrolled)



- PI control

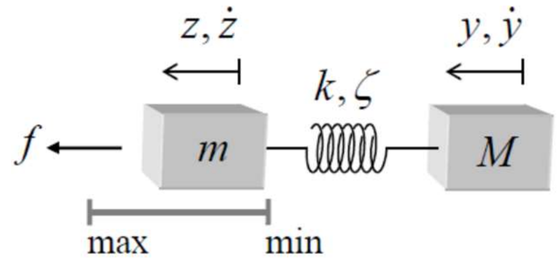


- PI + time-delay based



Why not observer-based compensation?

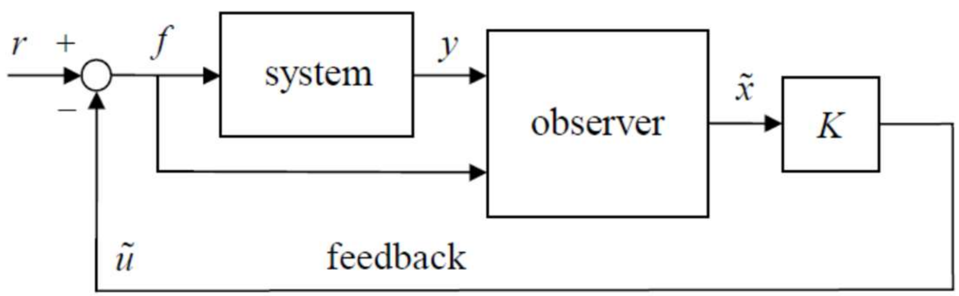
- When using state-feedback with observer



$$x \equiv (x_1, x_2, x_3, x_4)^\top = (\dot{z}, z, \dot{y}, y)^\top$$

$$\begin{aligned} \dot{x} &= A\,x + B\,f + D, \\ y &= C\,x, \end{aligned} \quad \Rightarrow \quad G(s) = C(sI - A)^{-1}B$$

$$e(t) = x(t) - \tilde{x}(t) \quad \Rightarrow \quad \dot{e}(t) = \tilde{A}e(t) = (A - QC)e(t)$$



$$\Rightarrow \quad \boxed{\dot{\tilde{x}} = (A - BK - QC)\tilde{x} + Bf + Qy}$$

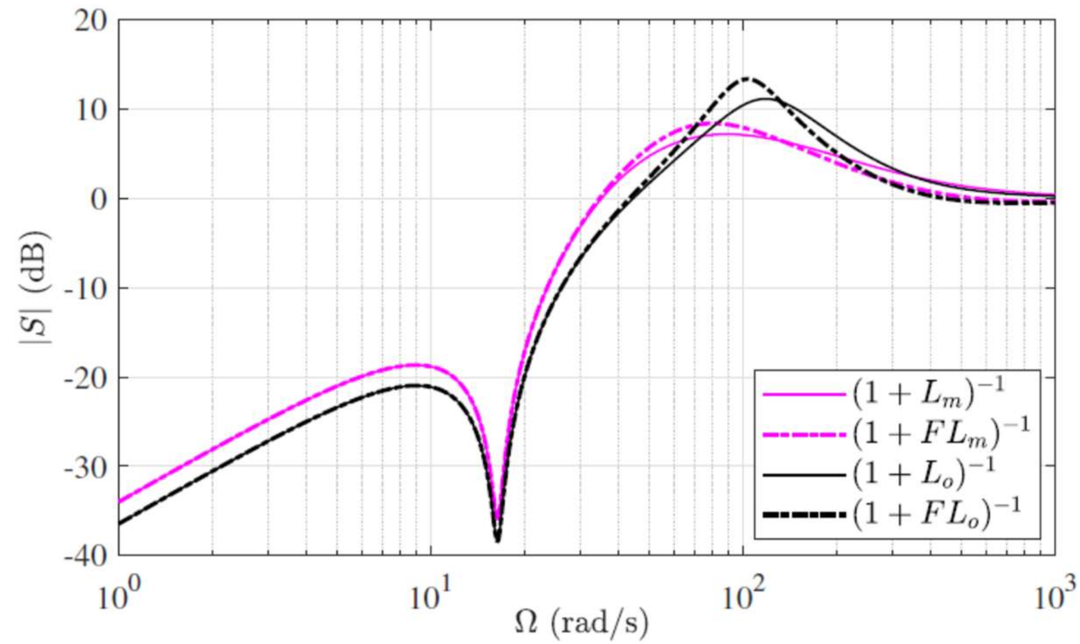
- Consider additional (practical) stability features, see [3]

loop transfer function from $r(s)$ to $\tilde{u}(s)$ $\quad L_o(s) = K\big(sI - A + BK + QC\big)^{-1}[B\,Q]\begin{bmatrix} 1 \\ G(s) \end{bmatrix}$

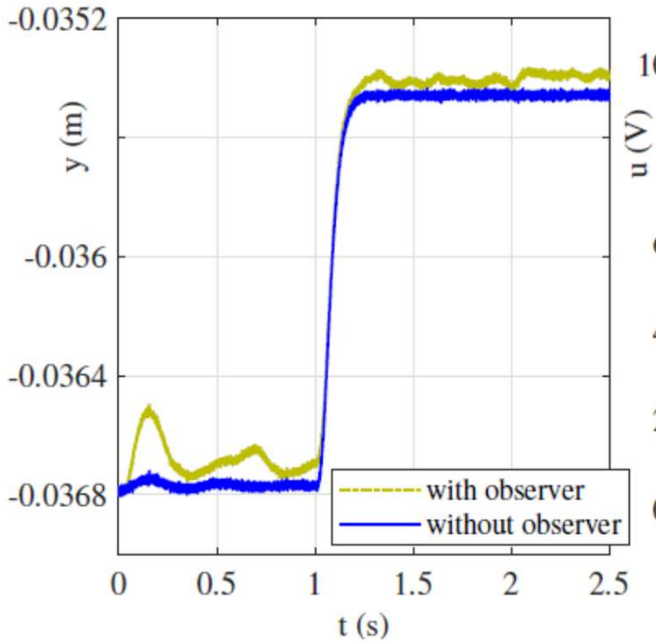
$$L_m(s) = K\big(sI - A\big)^{-1}B \quad \Rightarrow \quad \boxed{S_{\max} = \max_{\Omega}\big|S(i\Omega)\big| = \max_{\Omega}\Big|\big(1 + L(i\Omega)\big)^{-1}\Big|}$$

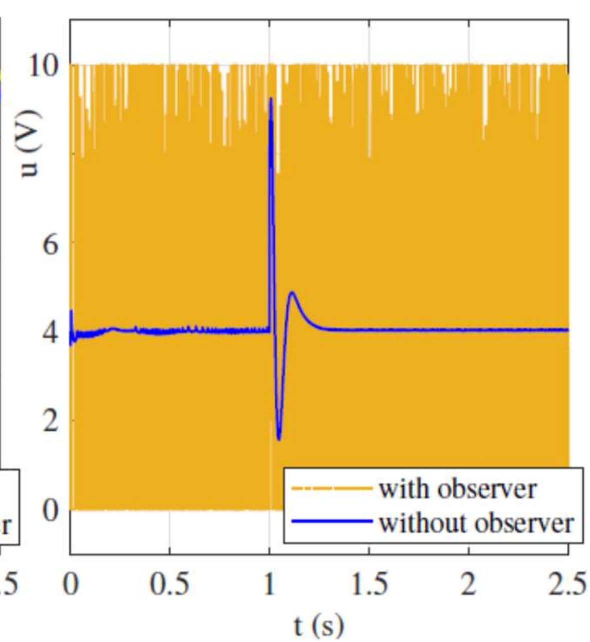
Why not observer-based compensation? (cont.)

- Low ‘stability margin’ S_{max} of observer-based feedback



- As consequence – unfeasible control behavior (even in the numerical simulations)





Adaptive extension of delay-based control

- Time-delay based feedback control [2] can be online tuned (i.e. be adaptive) by the robust frequency estimator [1], as shown in [3]

$$w(t) = y(t) - y\Big(t - \frac{\pi}{\beta}\Big), \quad \omega < \beta < 3\omega, \qquad (17)$$

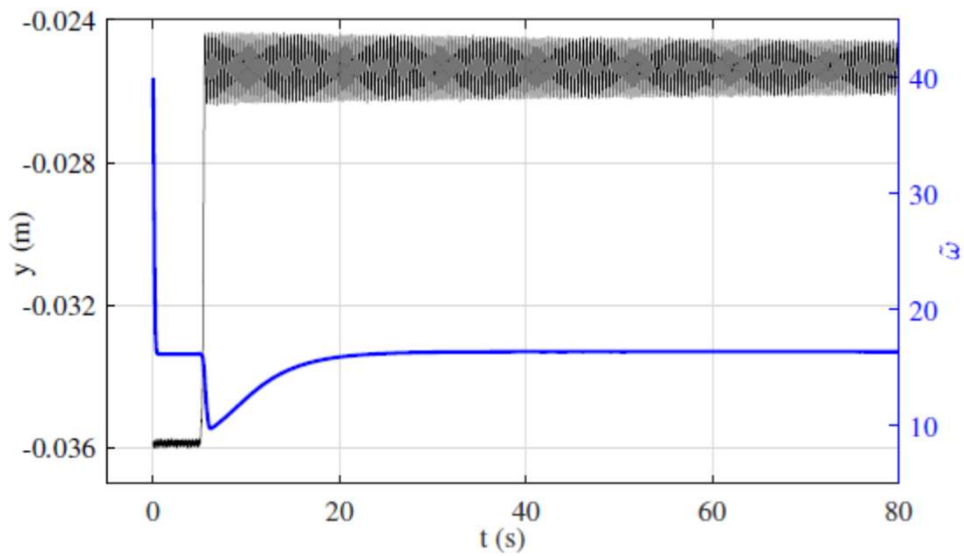
where β is a free adjustable time-delay parameter. Assuming a biased (by Y_0) harmonic oscillation

$$y(t) = Y_0 + Y \sin\big(\omega t + \phi\big), \qquad (18)$$

and substituting it into (17) results in

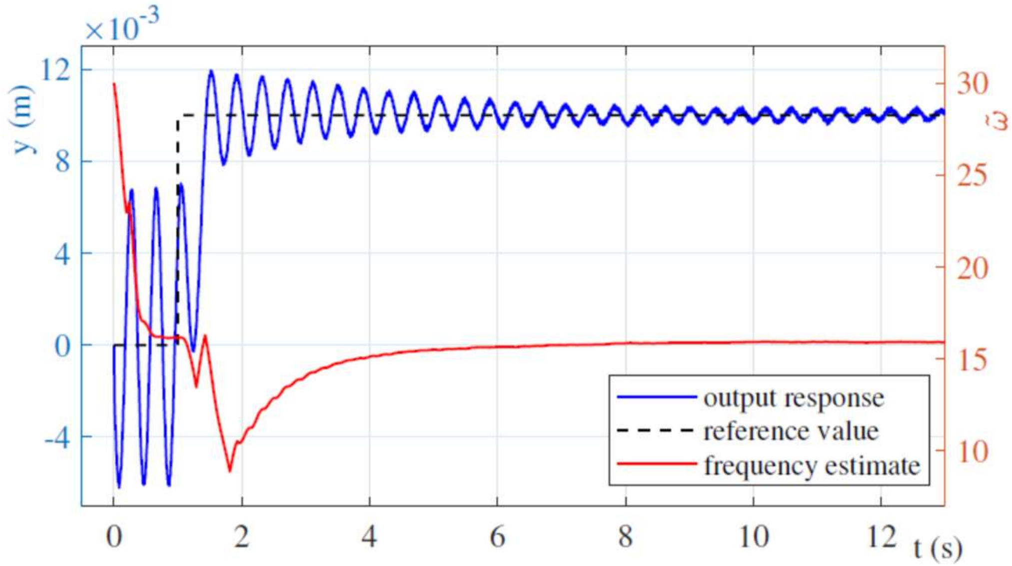
$$w(t) = Y\Bigg(\sin\big(\omega t + \phi\big) - \sin\Big(\omega t + \phi - \omega\frac{\pi}{\beta}\Big)\Bigg). \qquad (19)$$

- Converging oscillations frequency, even in presence of stepwise transients (here experiments)

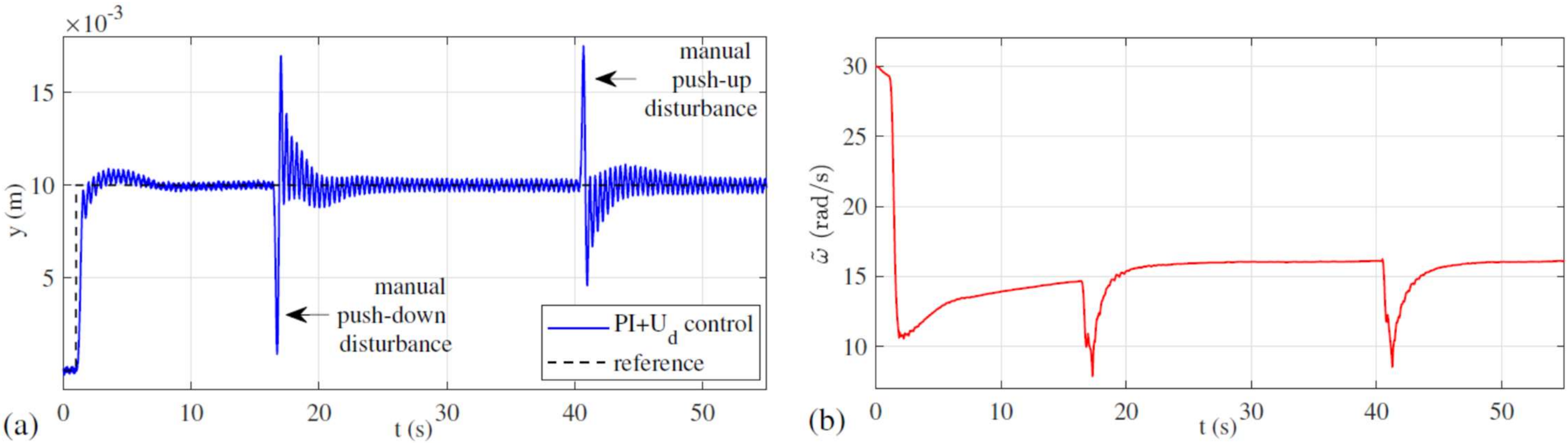


Adaptive extension of delay-based control (cont.)

- Experimental evaluation for oscillatory initial conditions



- Experimental evaluation with injected external disturbances, [3]



- # Previous related works and references

1. One-parameter robust global frequency estimator for slow-varying amplitude and noisy oscillations. M Ruderman. Mechanical Systems and Signal Processing, 170, pp. 108756, 2022
2. Time-delay based output feedback control of fourth-order oscillatory systems. M Ruderman. Mechatronics, 94, pp. 103015, 2023
3. Adaptive time delay based control of non-collocated oscillatory systems. M Ruderman. arXiv:2311.14979, http://arxiv.org/abs/2311.14979, 2023, to appear in IEEE MED, 2024
4. An adaptive notch filter for frequency estimation of a periodic signal. M. Mojiri, A.R. Bakhshai. IEEE Transactions on Automatic Control, 49 (2), pp. 314–318, 2004
5. A globally convergent frequency estimator. L. Hsu, R. Ortega, G. Damm. IEEE Transactions on Automatic Control, 44 (4), pp. 698–713, 1999
6. Stability and stabilization of systems with time delay. R. Sipahi, S.I. Niculescu, C.T. Abdallah, W. Michiels, K. Gu. IEEE Control Systems Magazine, 31(1), pp.38–65, 2011
7. Robust output feedback control of non-collocated low-damped oscillating load. M. Ruderman. In IEEE 29th Mediterranean Conference on Control and Automation (MED), pp. 639–644, 2021

Thank you for attention