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2024 MURPHYS

On structural hysteresis damping of vibro-impact dynamics: case study

Michael Ruderman

michael.ruderman@uia.no

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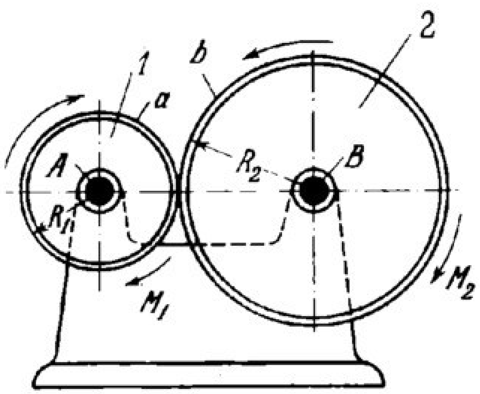
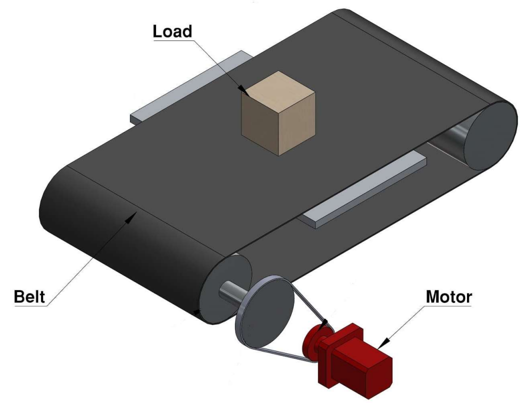
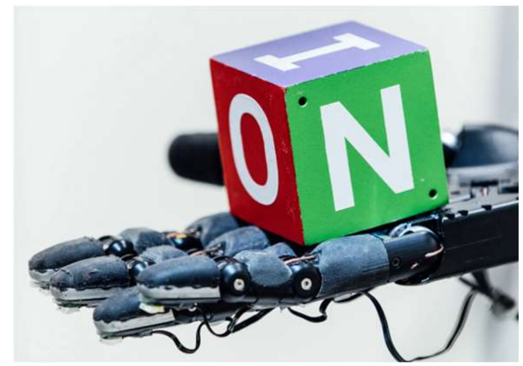
Benevento, Italy

Frictional interface of mechanical pair

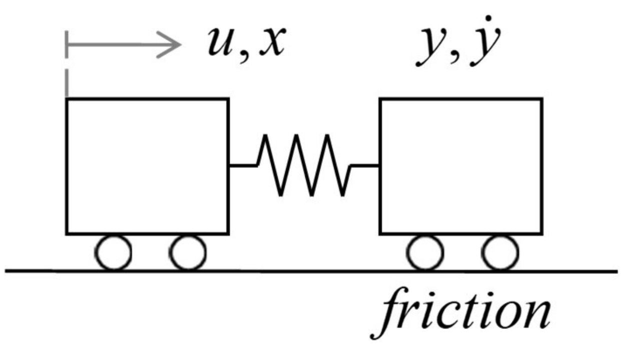
- Dynamics of **inertial pair** coupled **only** via **frictional interface**
- Various applications like, for instance, friction drives, items on conveyor belts and turntables, not-flanged manipulation in robotics, etc.



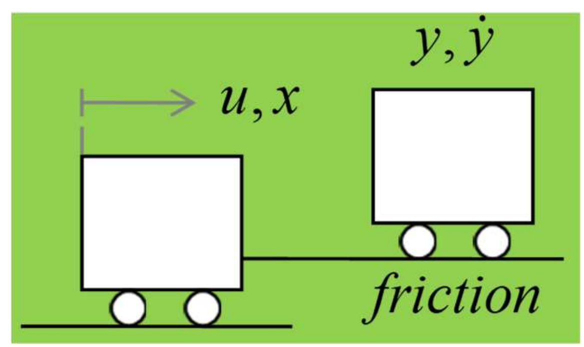




- Well defined solutions, also with discontinuities (i.e. in Filippov sense)

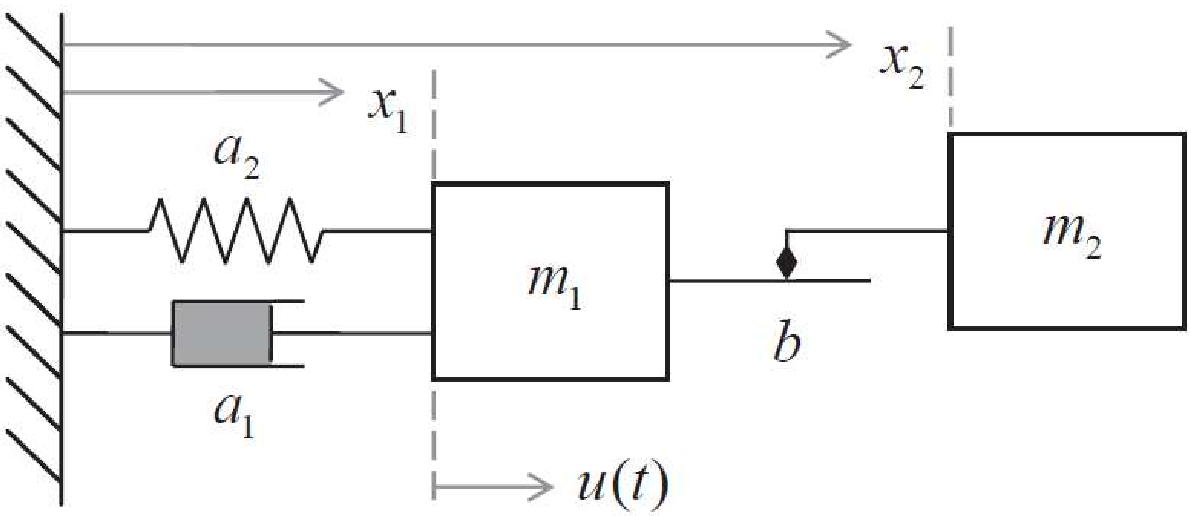


- Less studied dynamics, also with changed cause-effect relationship



Introduced modeling framework

- Driving inertial body m_1 and driven inertial body m_2 (in generalized coordinates)



[Ruderman, Zagvozdkin, Rachinskii, IEEE CDC, 2022]

- Coupling between two bodies via the Coulomb friction force

$$f = b\,\mathrm{sgn}(v), \quad v \equiv \dot{x}_1 - \dot{x}_2$$

$b > 0$: Coulomb friction coefficient

- Three-points signum function

$$\mathrm{sgn}(v) = \begin{cases} 1, & v > 0, \\ 0, & v = 0, \\ -1, & v < 0. \end{cases}$$

$\Rightarrow$ well-defined map for (physical) zero relative velocity

Introduced modeling framework (cont.)

- Proposed (new) dynamic model

$$x_1 - x_2 \quad =: \quad z, \qquad (2)$$

$$\Bigl(m_1 + m_2\bigl(1 - |\mathrm{sgn}(\dot z)|\bigr)\Bigr)\,\ddot x_1 + a_1 \dot x_1 + a_2 x_1 + b\,\mathrm{sgn}(\dot z) \quad = \quad u(t), \qquad (3)$$

$$m_2 \ddot x_1 \Bigl(1 - |\mathrm{sgn}(\dot z)|\Bigr)\frac{1}{2}\Bigl(1 - \mathrm{sgn}\bigl(|\ddot x_1| - b m_2^{-1}\bigr)\Bigr) - m_2 \ddot x_2 + b\,\mathrm{sgn}(\dot z) \quad = \quad 0. \qquad (4)$$

- <u>(i) mode</u>: m_2-body rests on m_1-body $(\dot x_1 = \dot x_2)$, and no switching $\left(|\ddot x_1| < b m_2^{-1}\right)$

$$\Rightarrow \quad \begin{aligned} &(m_1 + m_2)\ddot x_1 + a_1 \dot x_1 + a_2 x_1 = u \\ &m_2(\ddot x_1 - \ddot x_2) = 0 \end{aligned}$$

- <u>(ii) mode</u>: m_2-body slides over m_1-body $(\dot x_1 \neq \dot x_2)$

$$\Rightarrow \quad \begin{aligned} &m_1 \ddot x_1 + a_1 \dot x_1 + a_2 x_1 + b\,\mathrm{sgn}(\dot z) = u \\ &m_2 \ddot x_2 - b\,\mathrm{sgn}(\dot z) = 0 \end{aligned}$$

- switching from mode (i) to mode (ii): $\text{if } \ |\ddot x_1| > b m_2^{-1} \ \Rightarrow \ \ddot x_2 \neq \ddot x_1$
- switching from mode (ii) to mode (i): $\text{once } \ \dot x_1 = \dot x_2 \ \Rightarrow$ synchronization

Convergence

- Free system (2)-(4) without viscous damping (i.e. a_1=0) is equivalent to

$$\dot{x}_1 = v_1 \qquad (5)$$

$$m_1 \dot{v}_1 = -a_2 x_1 - b\,\mathrm{sign}(v_1 - v_2) \qquad (6)$$

$$m_2 \dot{v}_2 = b\,\mathrm{sign}(v_1 - v_2) \qquad (7)$$

- Assuming Lyapunov function candidate, which is energy function for (5)-(7)

$$E = \frac{1}{2} a_2 x_1^2 + \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

one obtains

$$\dot{E} = a_2 x_1 v_1 + m_1 v_1 \dot{v}_1 + m_2 v_2 \dot{v}_2 = \quad \ldots \quad \ldots \quad \ldots \quad = -b\left|v_1 - v_2\right| \le 0$$

- System dissipates energy if m_2 slides over m_1, i.e. $v_1 \neq v_2 \;\Rightarrow\; \dot{E} < 0$

and it conserves energy if m_2 rests on m_1, i.e. $v_1 = v_2 \;\Rightarrow\; \dot{E} = 0$

Convergence (cont.)

- For the intervals with *E*<0, that implies the system (2)-(4) is (locally) asymptotically stable on the intervals, consider the unforced dynamics of (3)

$$m_1 \ddot{x}_1 + a_1 \dot{x}_1 + a_2 x_1 = \pm b$$

- One can show that

$$\ddot{x}_1(t) = \frac{\mathrm{d}^2}{\mathrm{d}t^2}\left[\pm b \left(1 - \frac{1}{\sqrt{1-\zeta^2}} \exp(-\zeta\omega_n t) \sin\left(\omega_n \sqrt{1-\zeta^2}\, t + \phi \right) \right) \right]$$

where $\omega_n = \sqrt{\dfrac{a_2}{m_1}}, \quad \zeta = \dfrac{a_1}{2 m_1 \omega_n}, \quad \phi = \tan^{-1}\left(\dfrac{\sqrt{1-\zeta^2}}{\zeta} \right)$

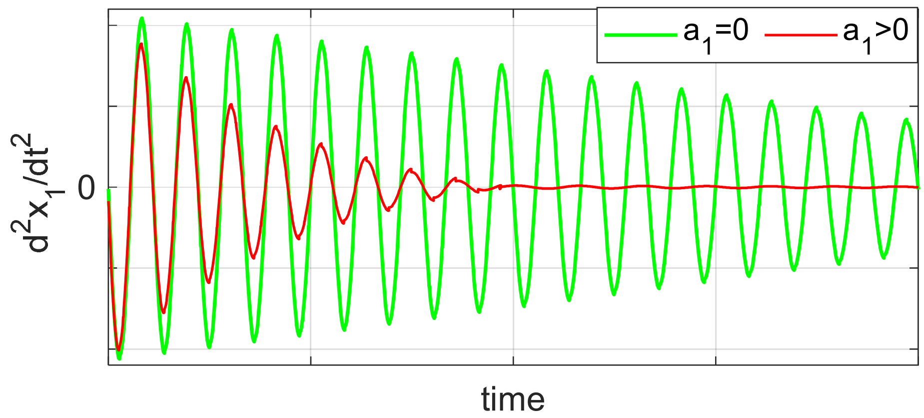
is continuously decaying on all intervals where *E*<0, that is in (ii)-mode

- Therefore, at some $T_s < t < \infty$

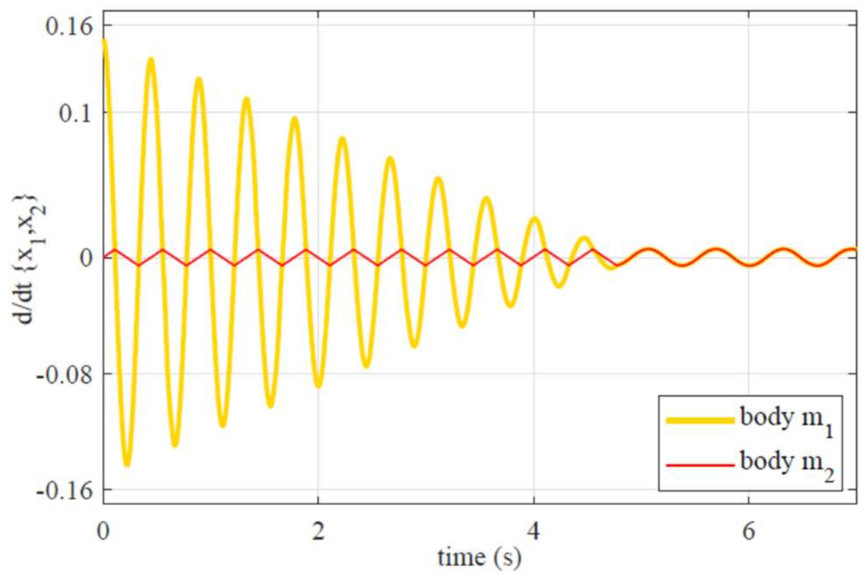
$|\ddot{x}_1(t)| < b\, m_2^{-1} \;\Rightarrow\;$ eq. (4) is no longer switching, i.e. (i)-mode perstains

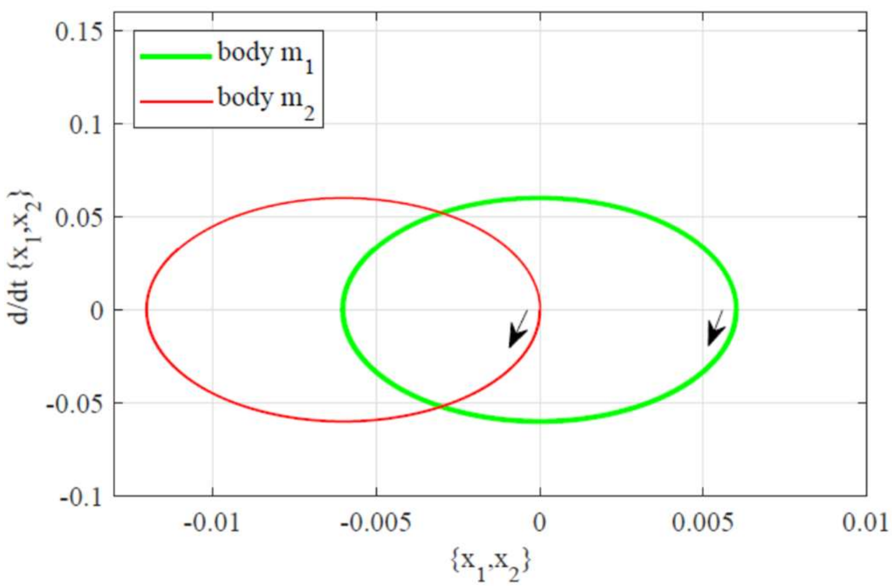
$$\Rightarrow \; \dot{x}_1(t) = \dot{x}_2(t) \quad \forall\, t > T_s$$

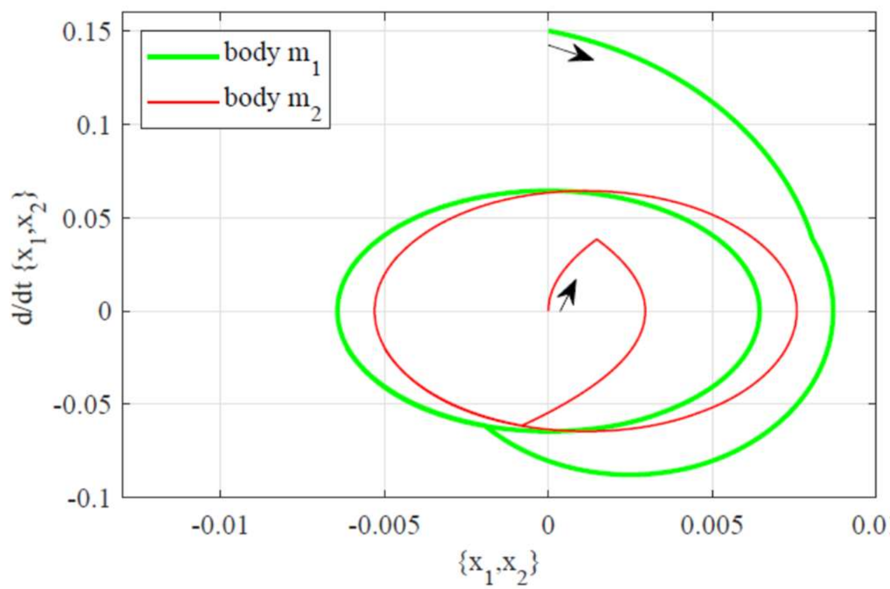
Numerical examples



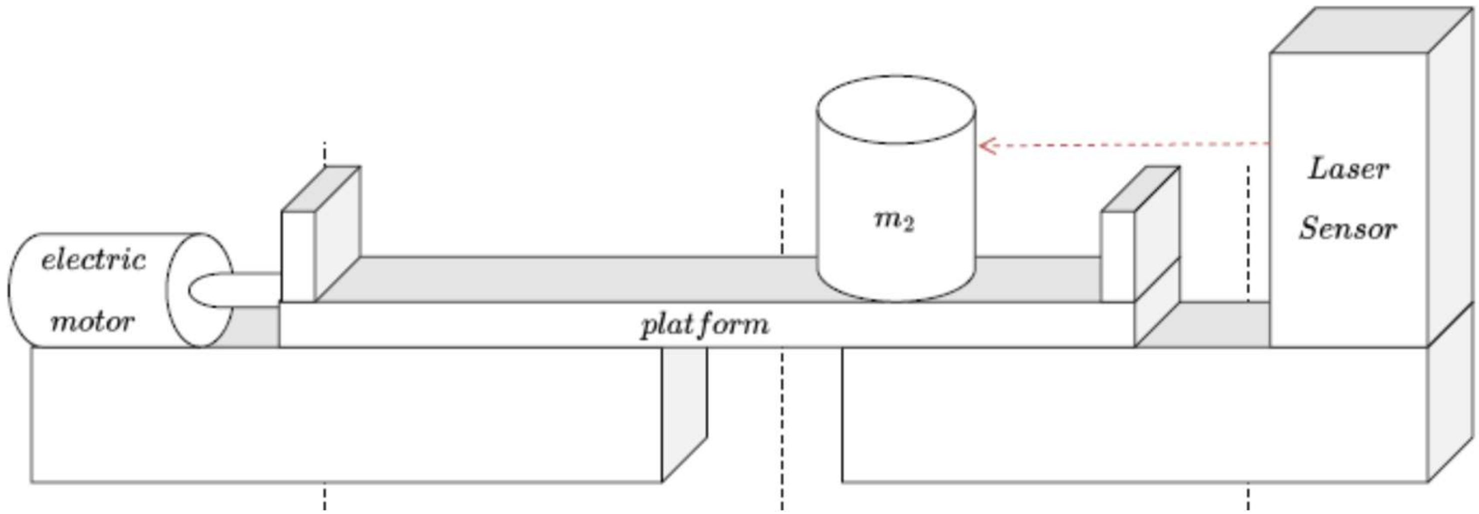
[Ruderman, Zagvozdkin, Rachinskii, IEEE CDC, 2022]





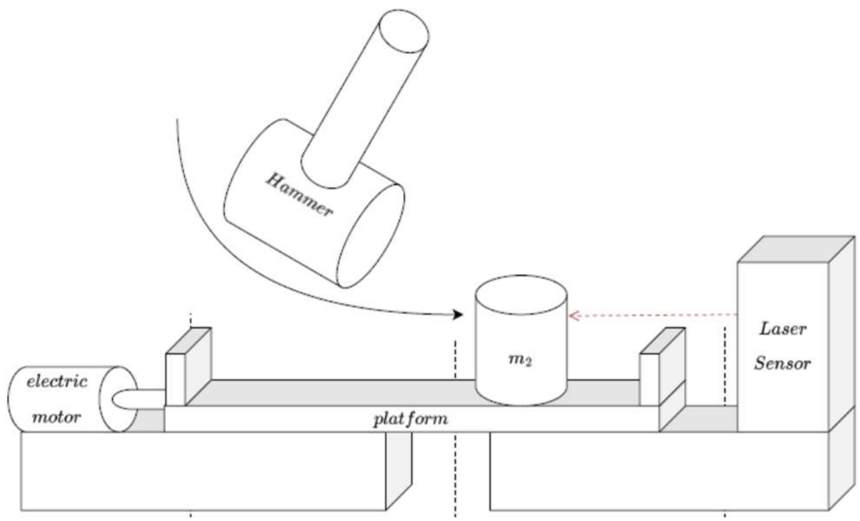


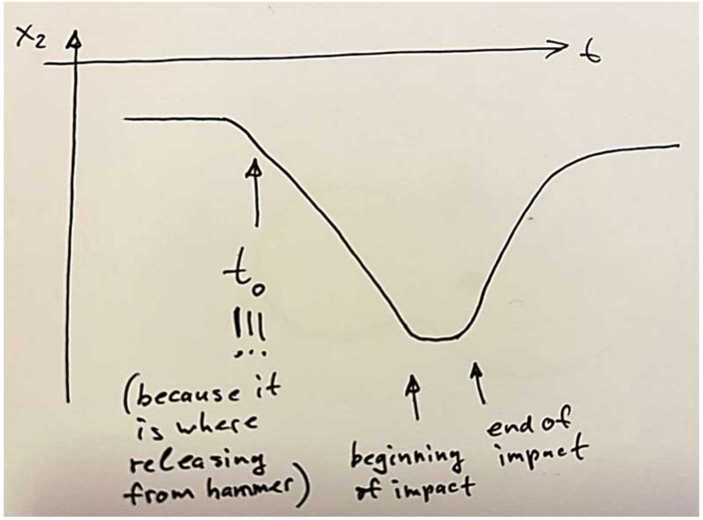
System with frictional interface and impact

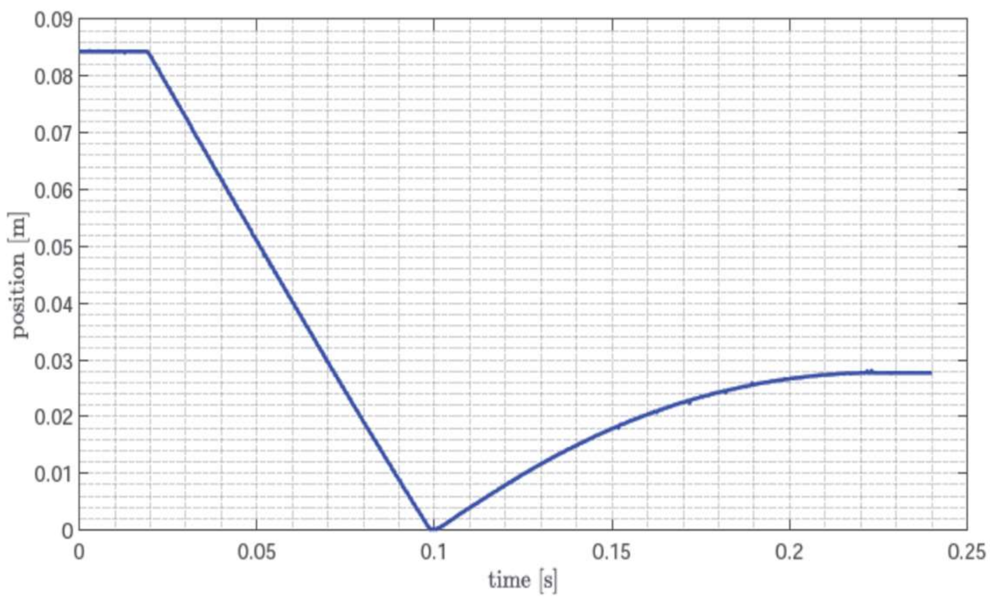


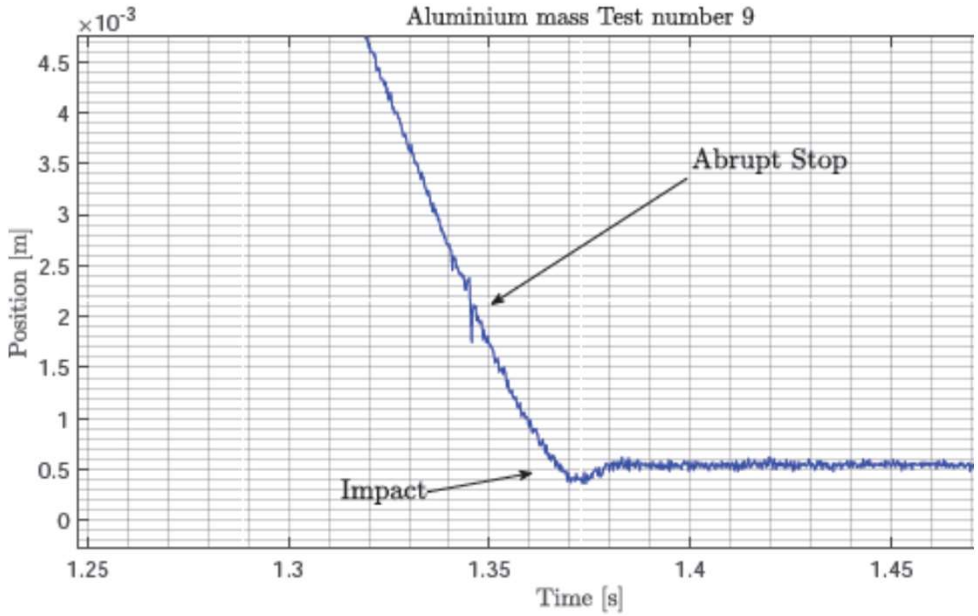
sampling rate	2 kHz
laser sensor repeatability	8 µm

Some observations from impact experiments

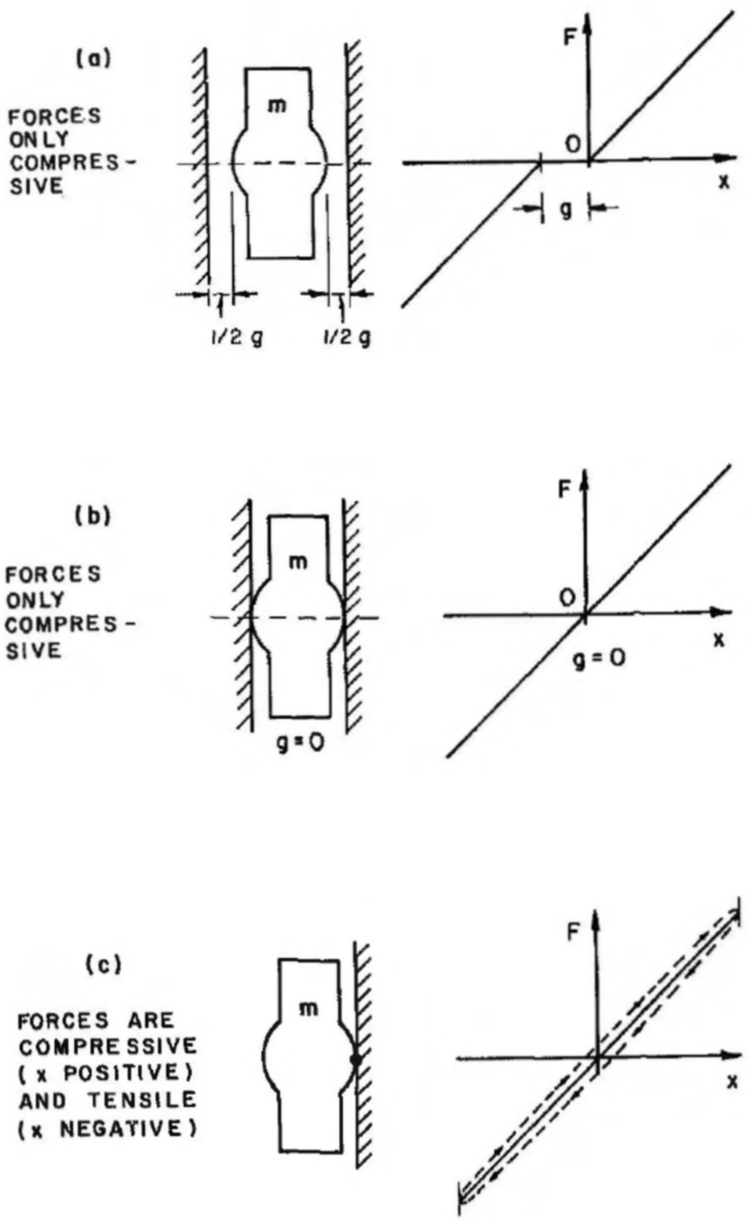






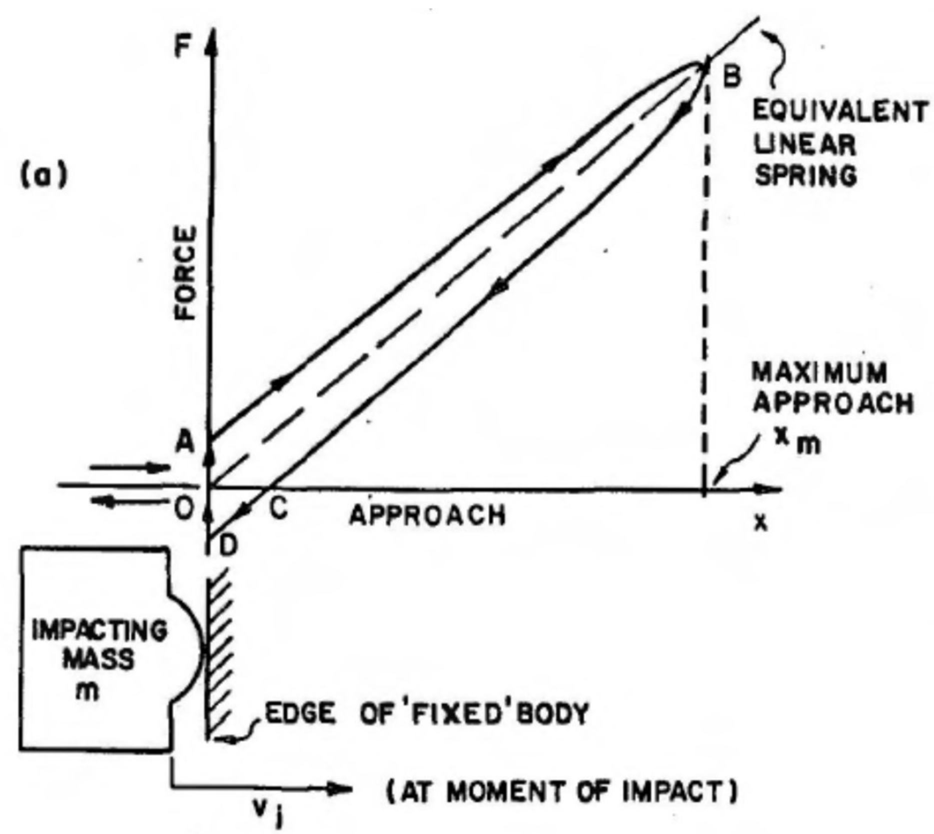


Problems in modeling vibroimpact



[Hunt, Crossley, Journal of App. Mechanics, 1975]

Spurious jumps in the contact force if allowing for a linear damping term

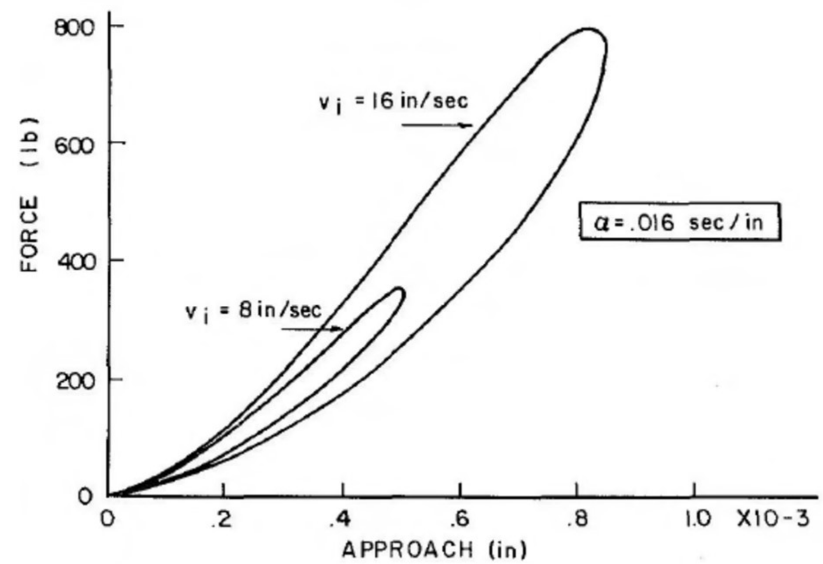


$$m\ddot{x} + c\dot{x} + kx = 0$$

Problems in modeling vibroimpact (cont.)

Overall nonlinear contact force

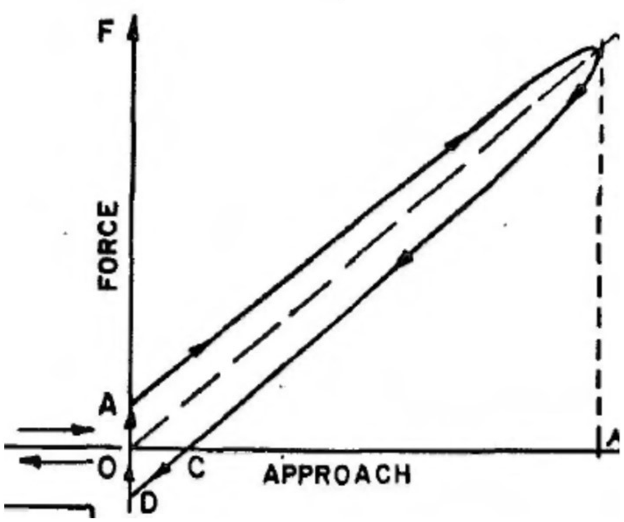
Closed hysteresis cycle of development of the contact force (here different impact velocities v_i)

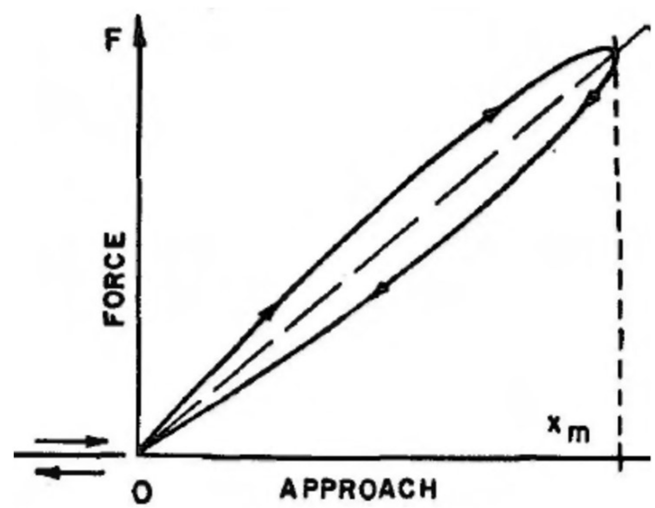


[Hunt, Crossley, Journal of App. Mechanics, 1975]

$$f = -b\dot{x} - kx$$

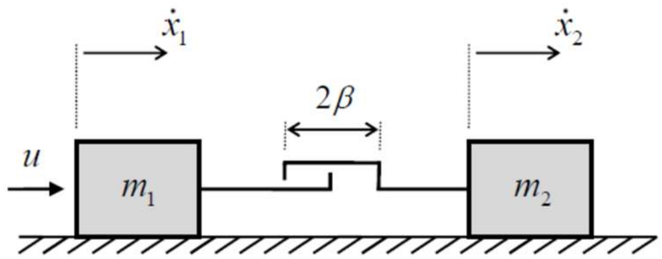
$$f = -(\lambda x^n)\dot{x} - kx^n$$

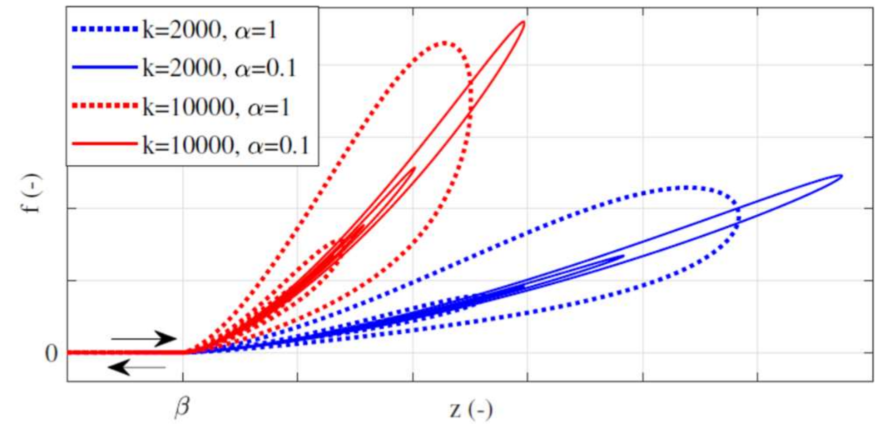




Mechanical backlash (as classical example)

[Ruderman, IEEE ISIE, 2021]
[Ruderman, Fridman, TCST, 2019]

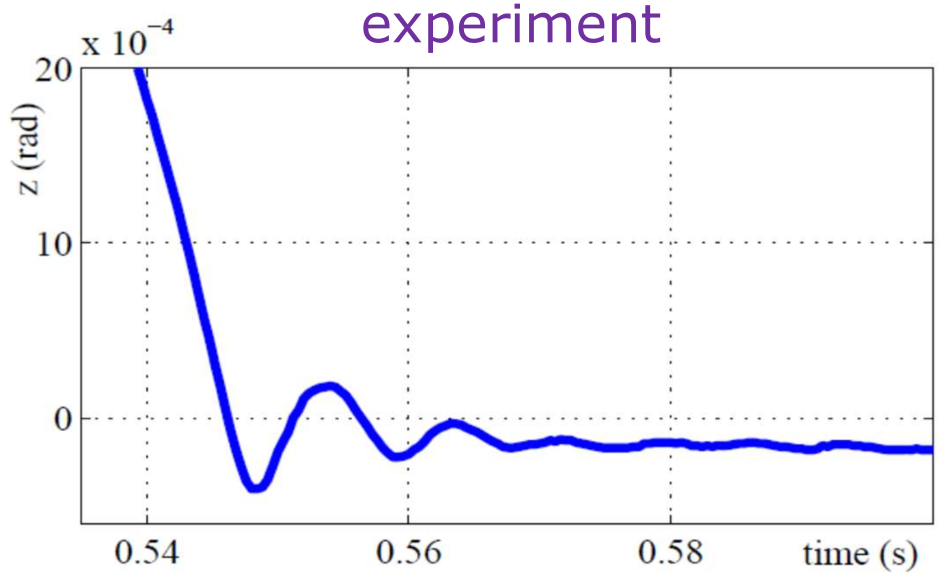




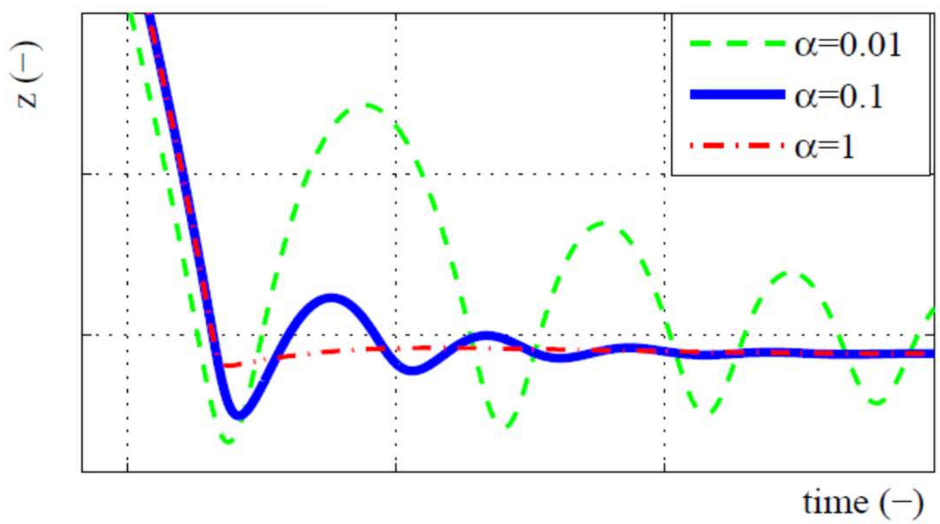
$$f = -(\lambda z^n)\dot{z} - kz^n \qquad \lambda = 1.5\alpha k$$

Parameter of the impacting surfaces *n*. For most simple assumption of the flat impacting surfaces *n*=1, while *n*=1.5 is consistent with Herzian theory of colliding spheres

experiment

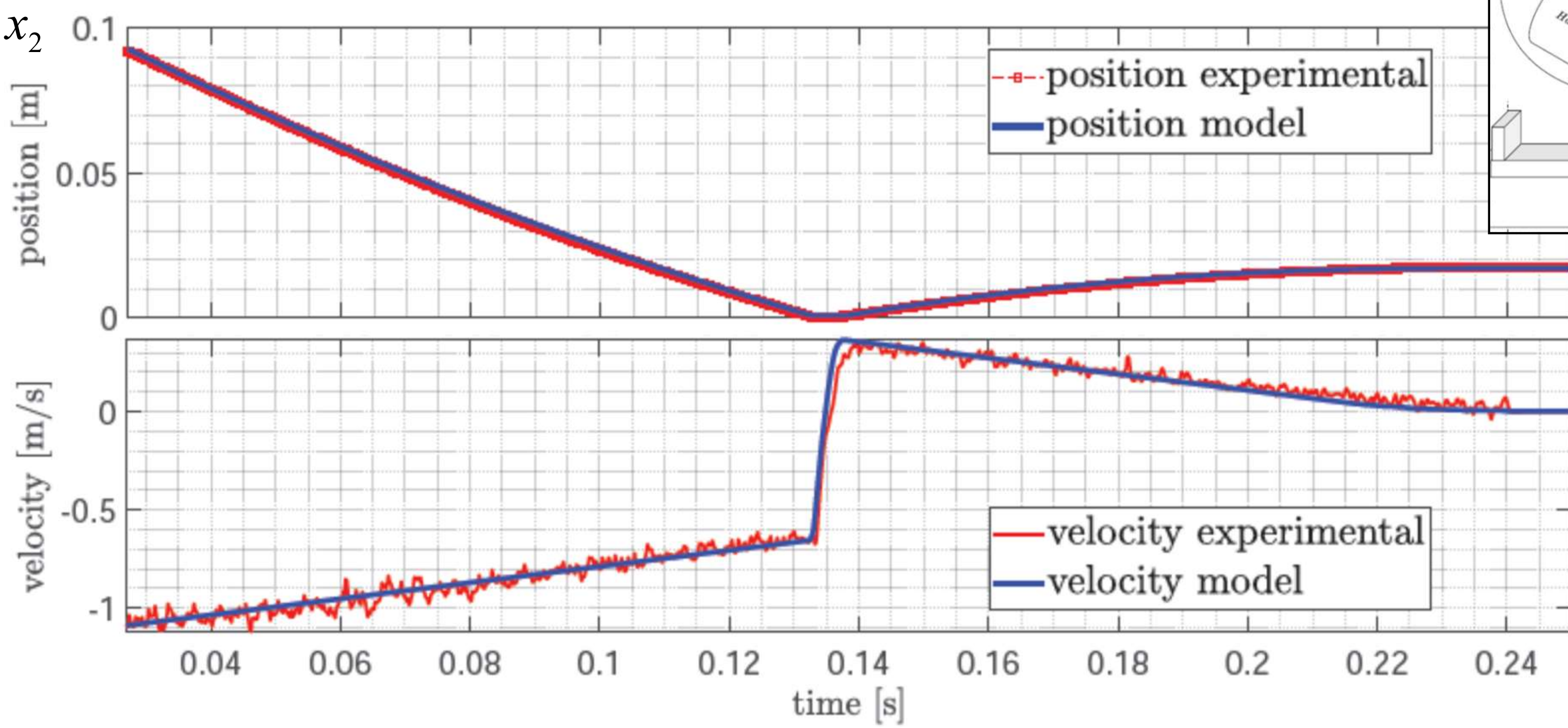


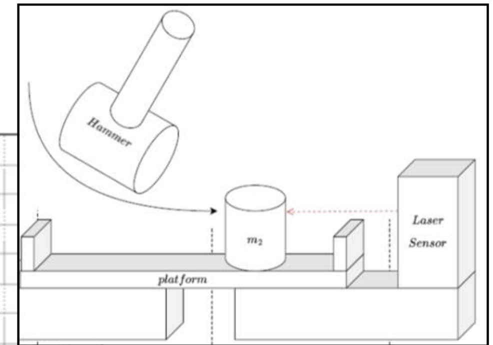
simulation



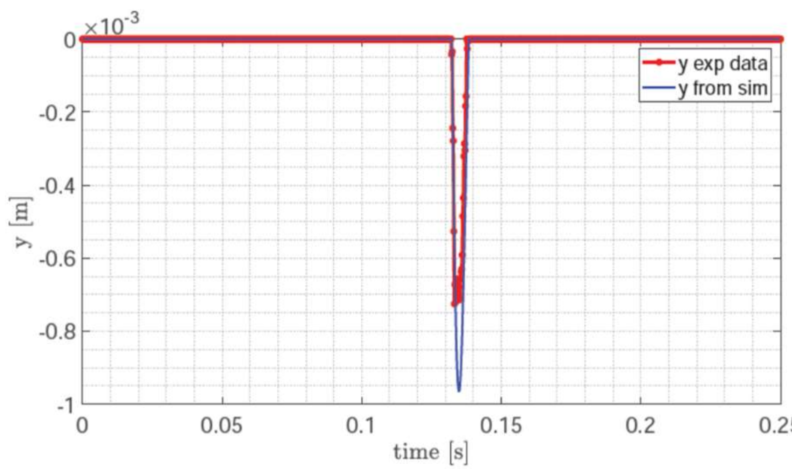
Hammer impact experiments

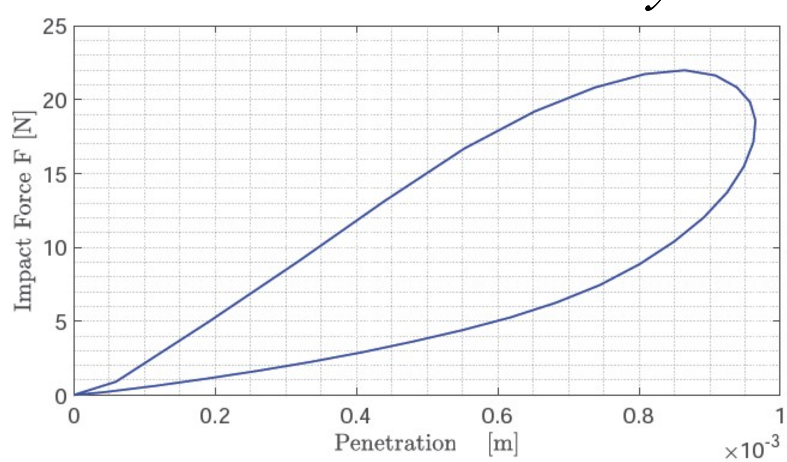
x_2





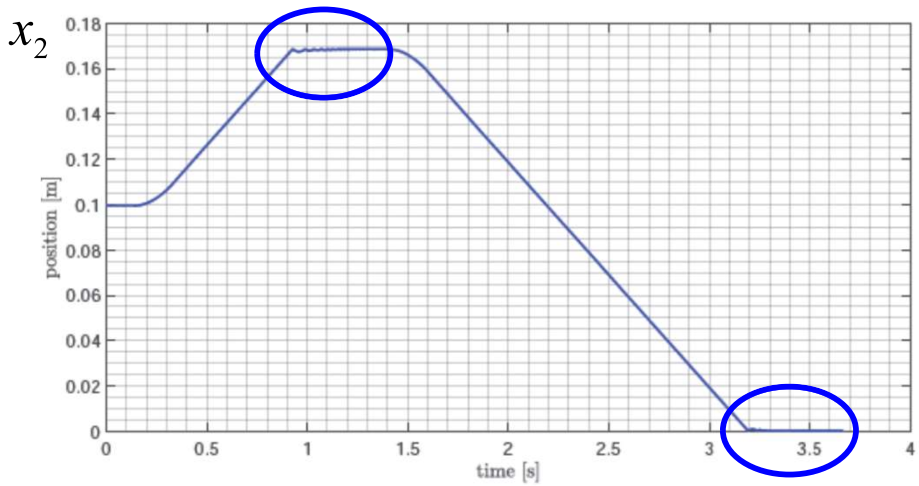
$$y = X_{impact} - x_2$$

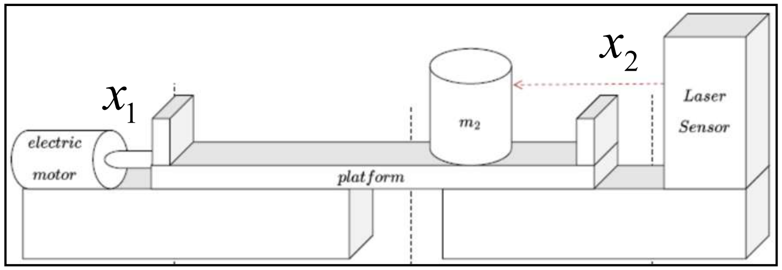


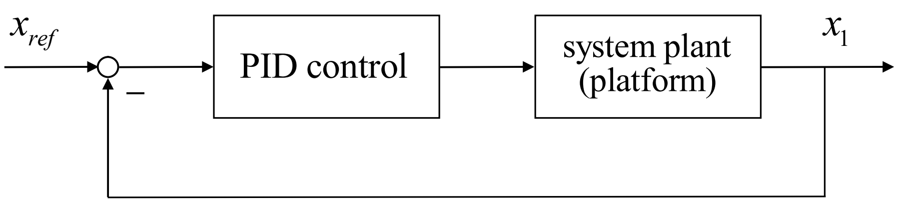


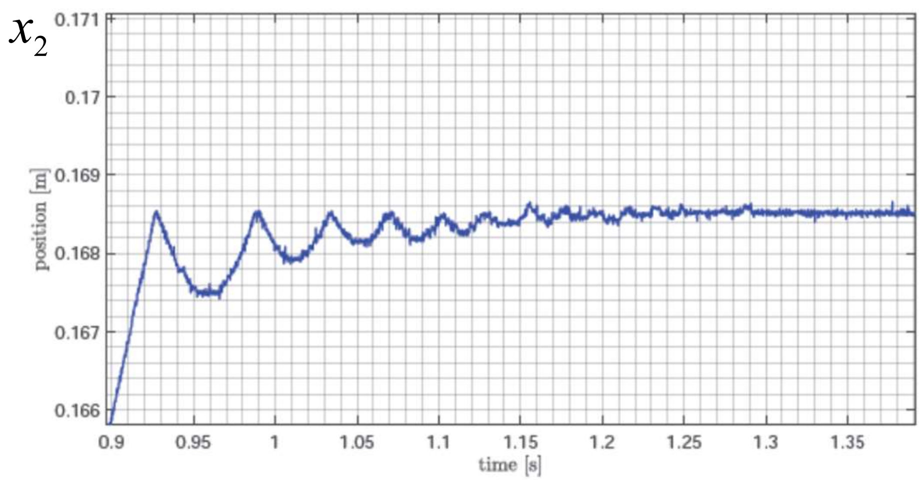
Experimental results (cont.)

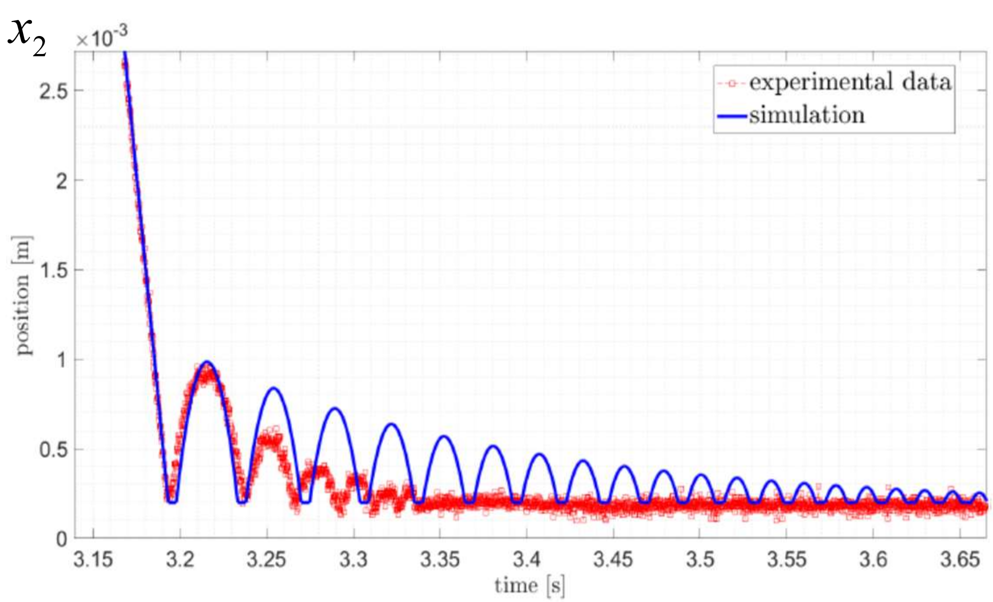
Constant velocity of the platform drive experiments (i.e. dx_1/dt = const)











Previous referred works

1. Dynamics of inertial pair coupled via frictional interface. M. Ruderman, A. Zagvozdkin, D. Rachinskii. IEEE 61th Conference on Decision and Control (CDC), 2022, pp. 1324-1328
2. On stiffness and damping of vibro-impact dynamics of backlash. M Ruderman. IEEE 30th International Symposium on Industrial Electronics (ISIE), 2021, p. 4
3. Model-free sliding-mode-based detection and estimation of backlash in drives with single encoder. M Ruderman, L Fridman. IEEE Trans. on Control Systems Technology 29 (2), 2019, pp. 812-817
4. Analysis and compensation of kinetic friction in robotic and mechatronic control systems. M. Ruderman, CRC Press, 1st edition, 2023
5. Coefficient of restitution interpreted as damping in vibroimpact. K. Hunt, F. Crossley. Journal of Applied Mechanics, vol. 42 (2), 1975, pp. 440-445

Thank you for attention

Grazie per l'attenzione

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