Disturbance Observer in Motion Control Applications

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Me, in short

Prof. Roberto Oboe
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Italy

- **I obtained the Laurea degree in Electrical Engineering in 1988 and the Doctorate degree from the University of Padova in 1992.**
- **From 1993 to 2003, I have been with the Dipartimento di Elettronica e Informatica of the University of Padova. From 2003 to 2008 I have been Associate Professor in Automatic Control at the Department of Mechanical and Structural engineering, University of Trento. Since 2008, I am Associate Professor of Automatic Control at the University of Padova, Dep.t of Management and Engineering (Vicenza).**
- **My interests are in the fields of Motion Control, Control and applications of MEMS devices, Applied Digital Control, Telerobotics, Virtual Mechanism, Haptic Devices, Biomedical Equipments.**
- **I am author/coauthor of more than 200 contributions, plus 6 international patents.**
- **I am a Fellow of the IEEE and, for the Industrial Electronics Society (IES), I served as VP for Planning and Development, VP for Technical Activities, Chair of the Technical Committee on Motion Control and Chair of the Management Committee of the IEEE/ASME Transactions on Mechatronics. I am currently member of the Committees on Senior and Life AdCom Member, Multimedia, Technical Activities Committee, Publications. I am also Co-EIC of the IEEE Transactions on Industrial Electronics and Associate Editor for the OJIES.**

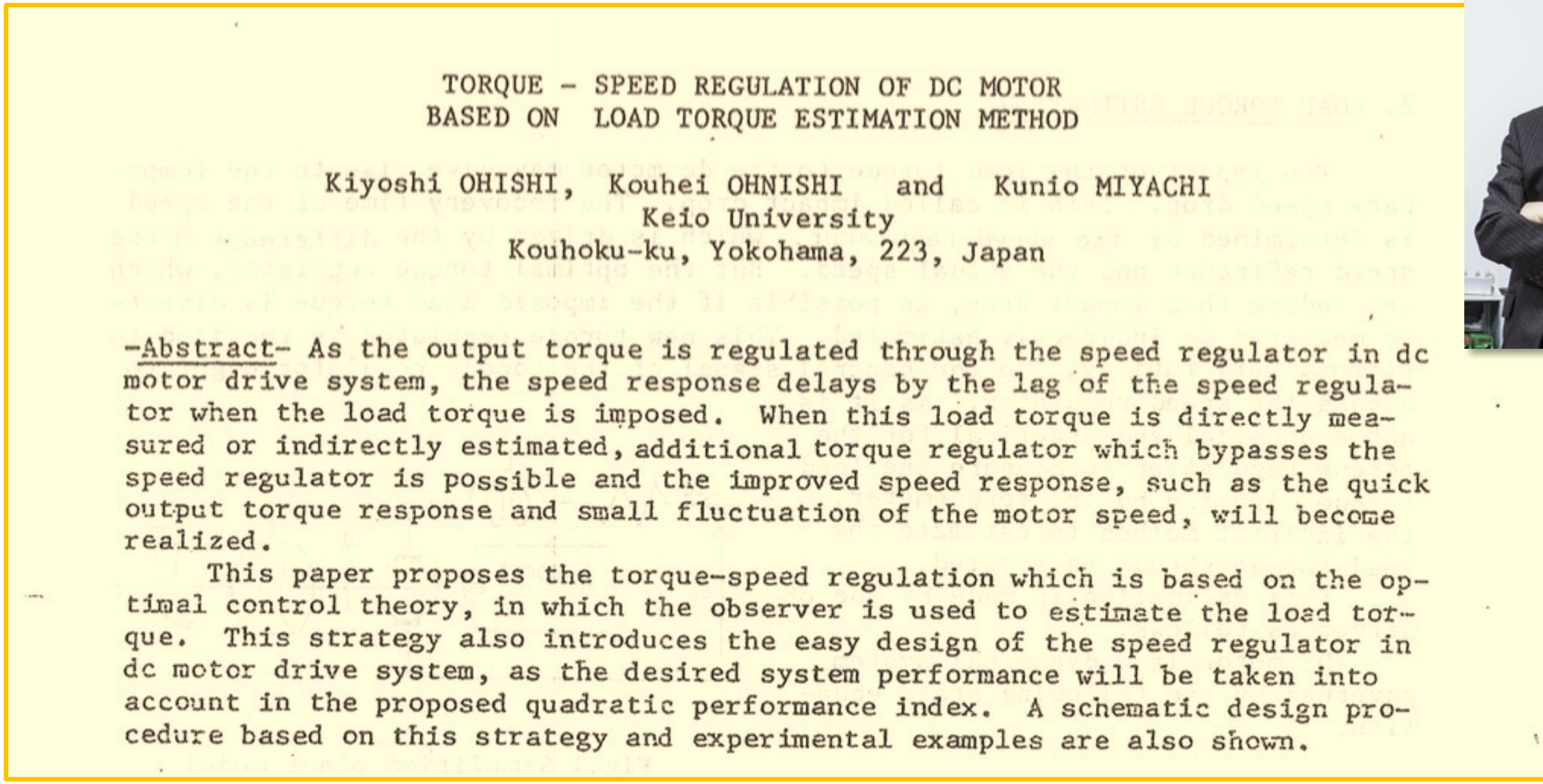
- Several years ago, a research and education program on Mechatronics has been started in the Vicenza Branch of the University of Padova

Mechatronics in Vicenza

- Vicenza has about 3000 Engineering Students
- 70 Faculties, 50 PhD & Post-docs, 30 Adm. & Tech.
- 15 Laboratories
- 3 bachelor and 3 master degrees
  - Management, Mechatronics, Product Innovation
- 2 Doctoral schools
  - Mechatronics, Management
- Erasmus flow with Grimstad (thesis/research)

A bit of history

- DOB has been proposed by Prof. Ohnishi, at IPEC Tokyo, 1983



TORQUE – SPEED REGULATION OF DC MOTOR
BASED ON LOAD TORQUE ESTIMATION METHOD

Kiyoshi OHISHI, Kouhei OHNISHI and Kunio MIYACHI
Keio University
Kouhoku-ku, Yokohama, 223, Japan

-Abstract- As the output torque is regulated through the speed regulator in dc motor drive system, the speed response delays by the lag of the speed regulator when the load torque is imposed. When this load torque is directly measured or indirectly estimated, additional torque regulator which bypasses the speed regulator is possible and the improved speed response, such as the quick output torque response and small fluctuation of the motor speed, will become realized.

This paper proposes the torque-speed regulation which is based on the optimal control theory, in which the observer is used to estimate the load torque. This strategy also introduces the easy design of the speed regulator in dc motor drive system, as the desired system performance will be taken into account in the proposed quadratic performance index. A schematic design procedure based on this strategy and experimental examples are also shown.

A bit of history

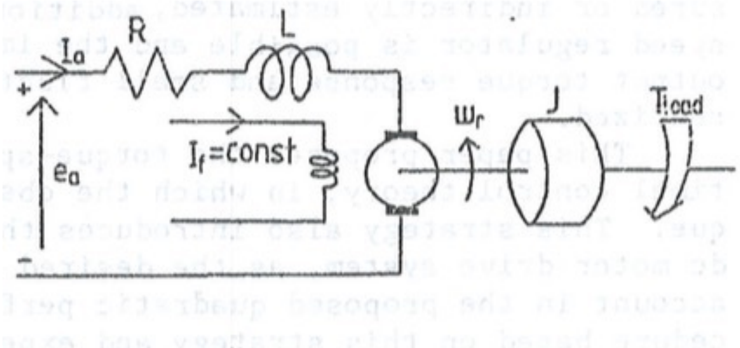
- The initial approach was aimed at estimating and compensating the load torque, under the assumption that model parameters were known
- 

Fig.1 Simplified plant model of dc motor

$$\frac{d}{dt}\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}e_a + \mathbf{E}T_{load}$$

$$\mathbf{y} = \mathbf{c}\mathbf{x}$$

$$\mathbf{x} = \begin{bmatrix} I_a \\ \omega_r \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} -\frac{R}{L} & -\frac{K_e}{L} \\ \frac{K_e}{J} & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} 0 \\ -\frac{1}{J} \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A bit of history

- Using a zero-order model of the unknown load torque, it was possible to set up a reduced-order estimator

$$\hat{T}_{load} = \frac{1}{s + a}\left[b I_a + \{ c - d(s + a) \} \omega_r \right]$$

$$\hat{T}_{load} = \frac{1}{1 + \tau s} T_{load}$$

$$a = \frac{1}{\tau}, \quad b = \frac{K_e}{\tau}, \quad c = \frac{J}{\tau^2}, \quad d = \frac{J}{\tau}$$

A bit of history

- The resulting implementation diagram looks familiar

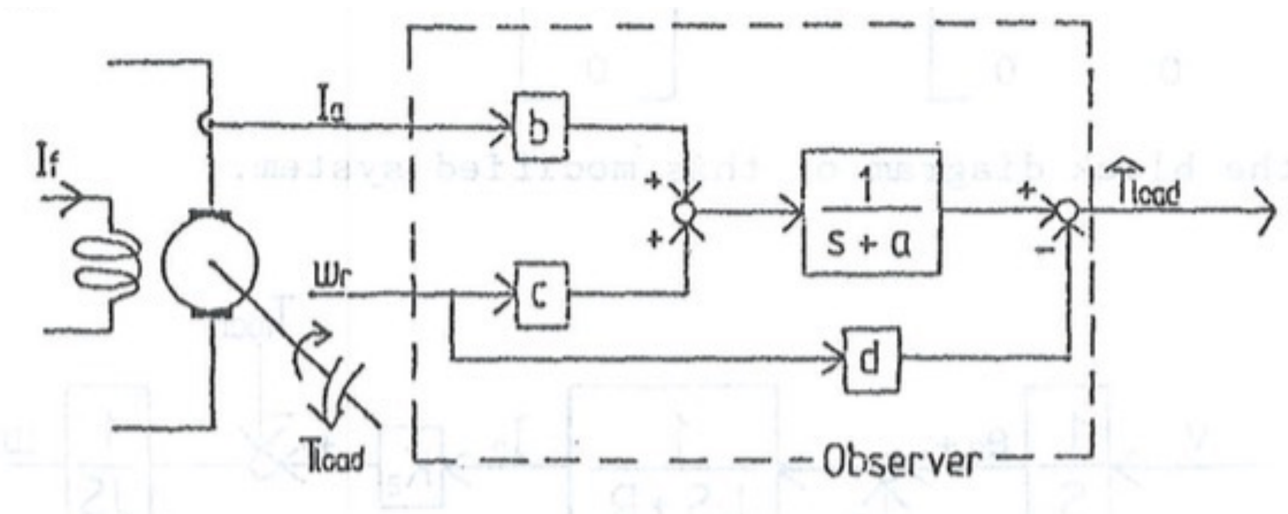
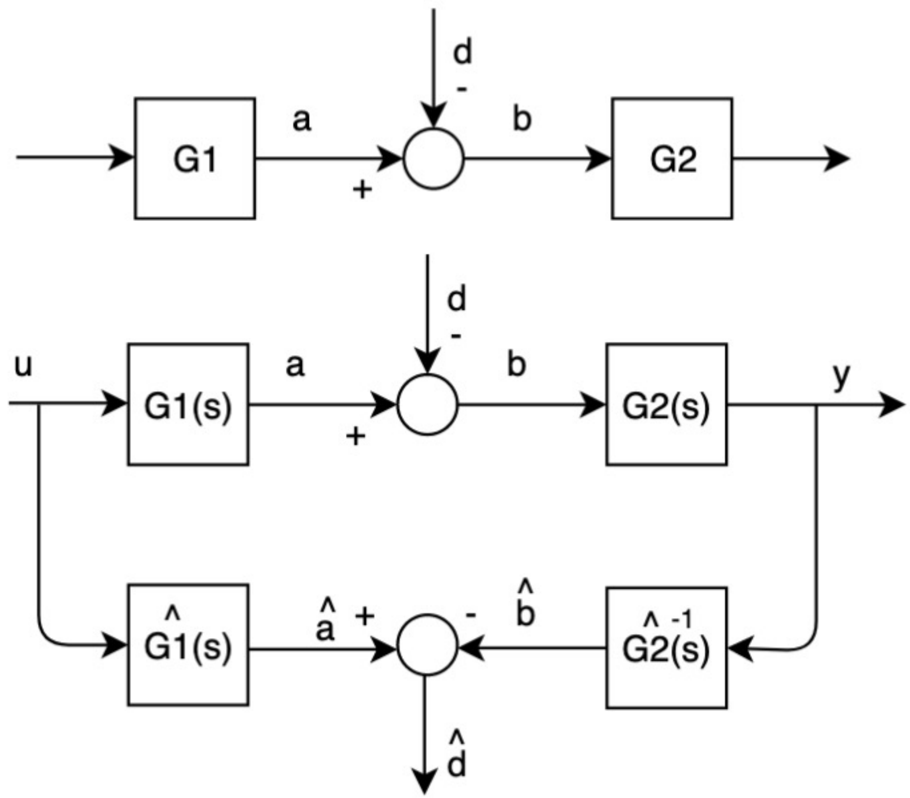
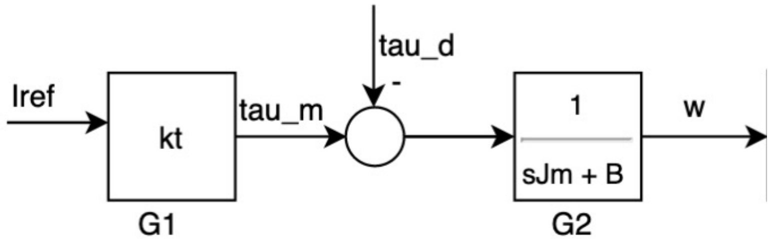


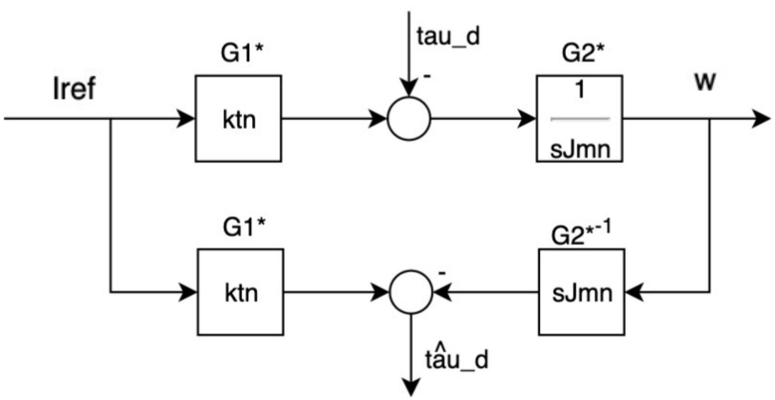
Fig.2 Schematic structure of load torque observer

- In fact, this is an equivalent approach for building the observer:



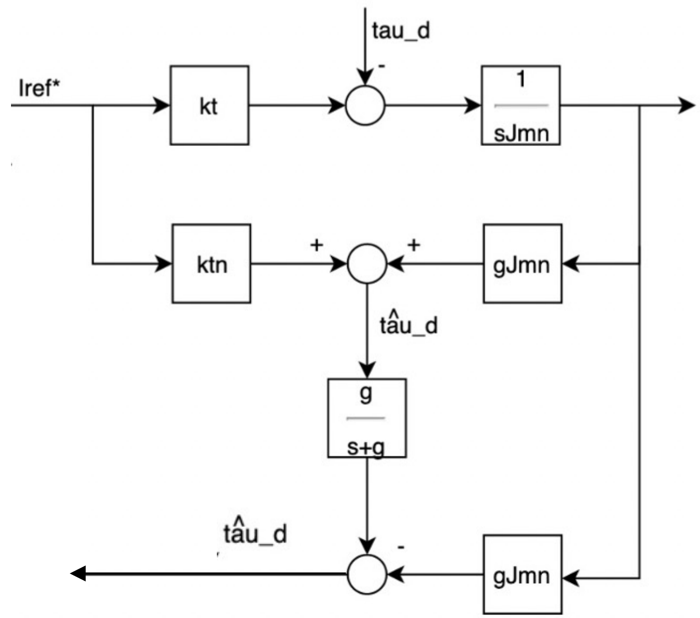
- Applying the concept to the motor…



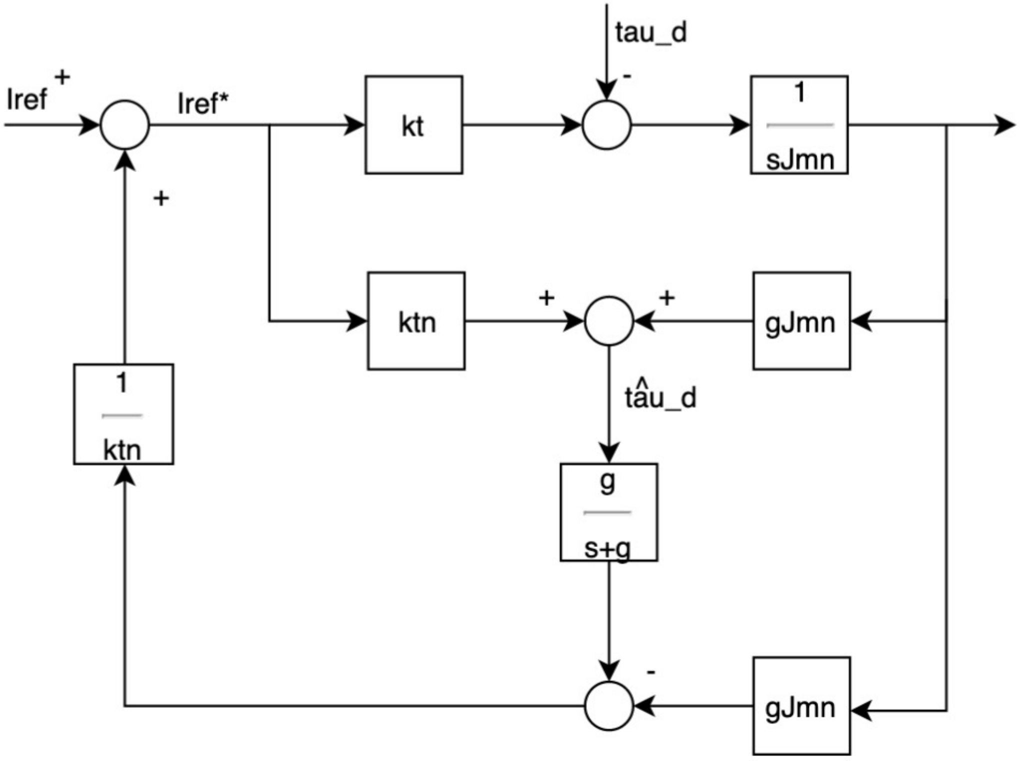


$$\hat{T}_{dis}(s) = K_{tn}I_{ref}(s) - sJ_{mn}\Omega(s)$$

Feasible version…

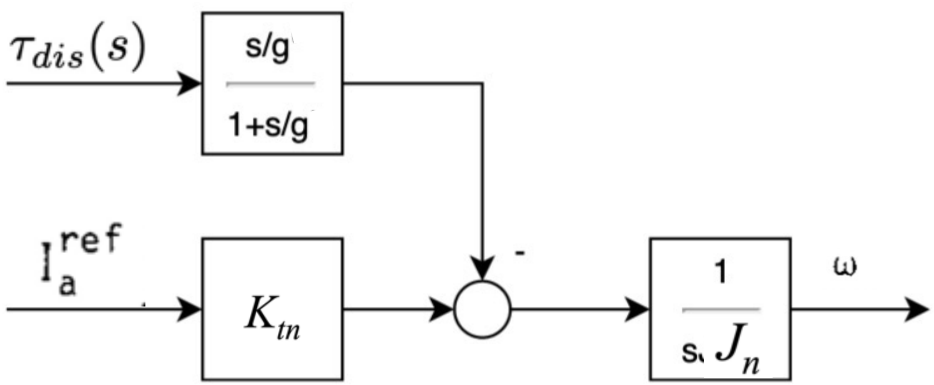


- The estimated disturbance can be used in compensating the actual one



A bit of history

- After some manipulations, it can be shown that it is equivalent to this system

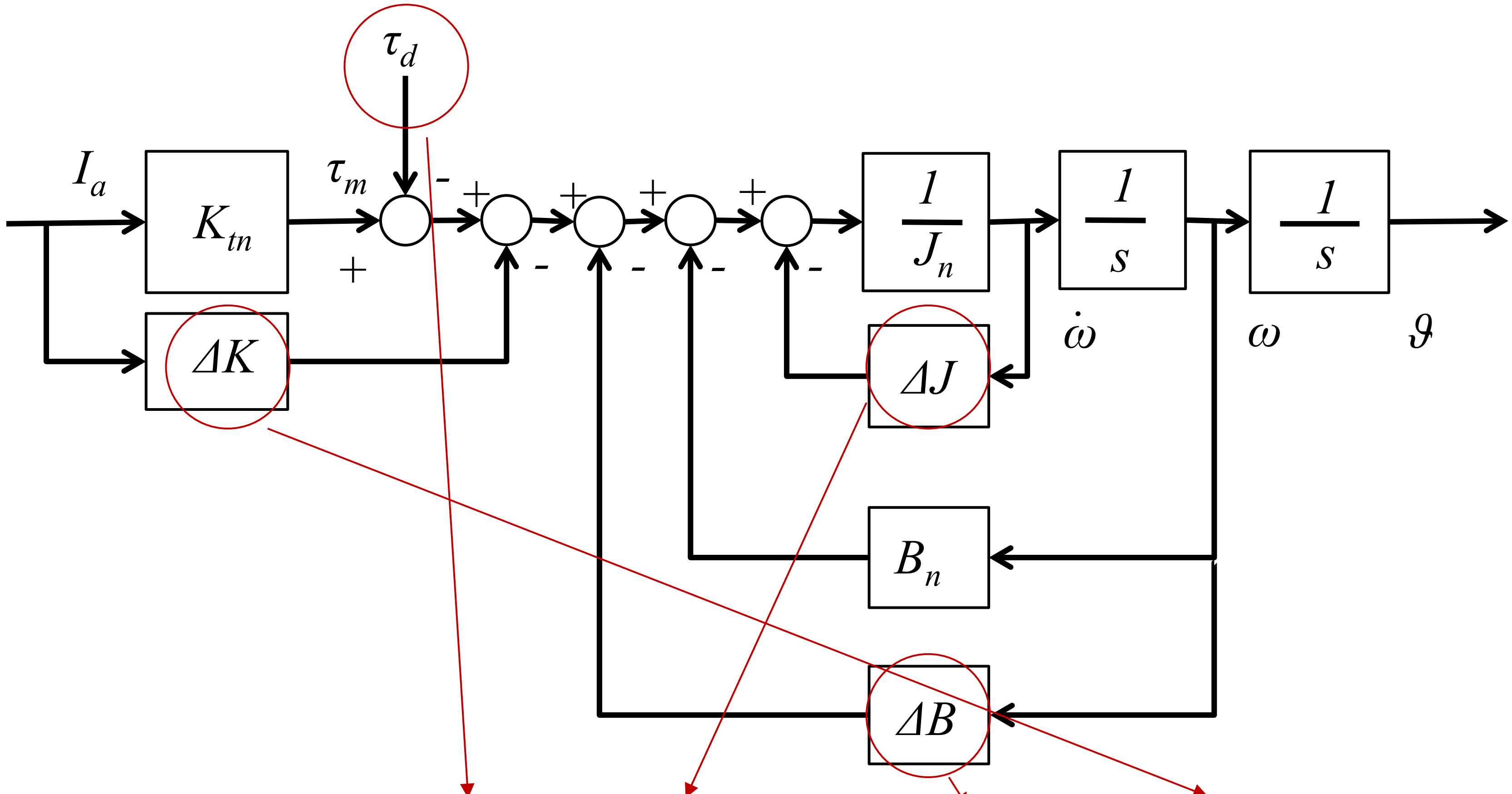


A bit of history

- But this was only the beginning…
- Ohnishi further developed the concept and realized that the estimator could have been designed on nominal parameters, instead of actual ones
  - He introduced the concept of "equivalent disturbance"
  - It contains not only the actual load torque, but also the effects of differences between nominal and actual plant

$$J = J_n + \Delta J; B = B_n + \Delta B; K_t = K_{tn} + \Delta K$$

A bit of history



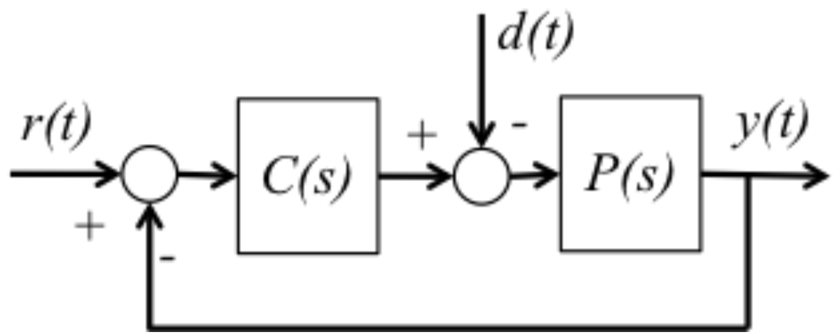
$$\tau_{dis}(s) = \tau_d(s) + \Delta J s \Omega(s) + \Delta B \Omega(s) - \Delta K I_a(s)$$

A bit of history

- In other words, the compensating signal $I_{cmp} = \tau_{dis}(s)/K_{tn}$, obtained from the estimated equivalent disturbance, implements the cancellation of some portion of the effects of the latter, leading to a system characterized by an ”equivalent disturbance–free” dynamics,
  - i.e. the compensated system behaves like one, with nominal parameters and zero disturbances, at least when such disturbances are slow enough.
  - Some had the feeling that the DOB was nothing but another way to implement an integral action in the control, to get a disturbance rejection

PID vs. DOB

- It is well known from the standard Automatic Control courses that a common approach for the rejection of the disturbances acting at the system's input is to have a very high gain of the controller, at least in the frequency range where the disturbance exerts its action.
  - In this way, the controller generates a compensating signal, while keeping the the error between reference *r(t)* and actual output *y(t)* at a small (or even zero) value

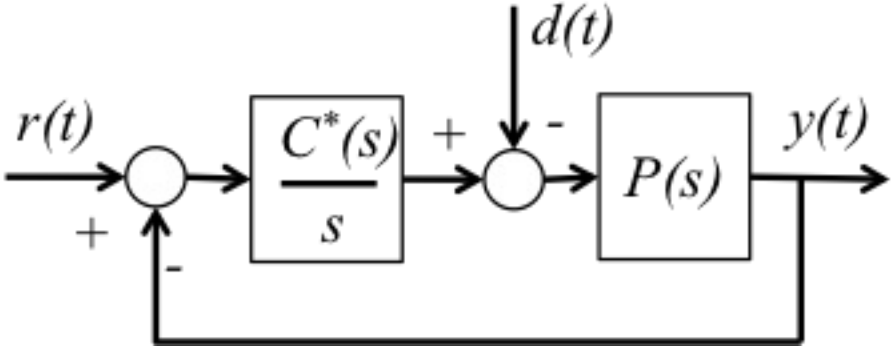


PID vs. DOB

- Unless the disturbance is known in its limited frequency range, however, such a tailored design of the controller is not feasible and a more simplistic approach is used.
  - In practice, an integral action is embedded in the controller, so that its gain tends to infinity as the frequency goes to zero.
  - With this solution, complete rejection of the constant disturbances is achieved, while a often satisfactory performance against time–varying ones can be achieved, with an appropriate design of the overall controller.

PID vs. DOB

- What the DOB promises is to have the same rejection of disturbances and, for this reason, when it has been proposed, many researchers labeled it as an alternative implementation of a standard integral action.
  - Let's briefly see whether this is the case or not
- We can consider a controller $C(s)$ factorised as $C^*(s)/s$, to consider the presence of a single integral action.



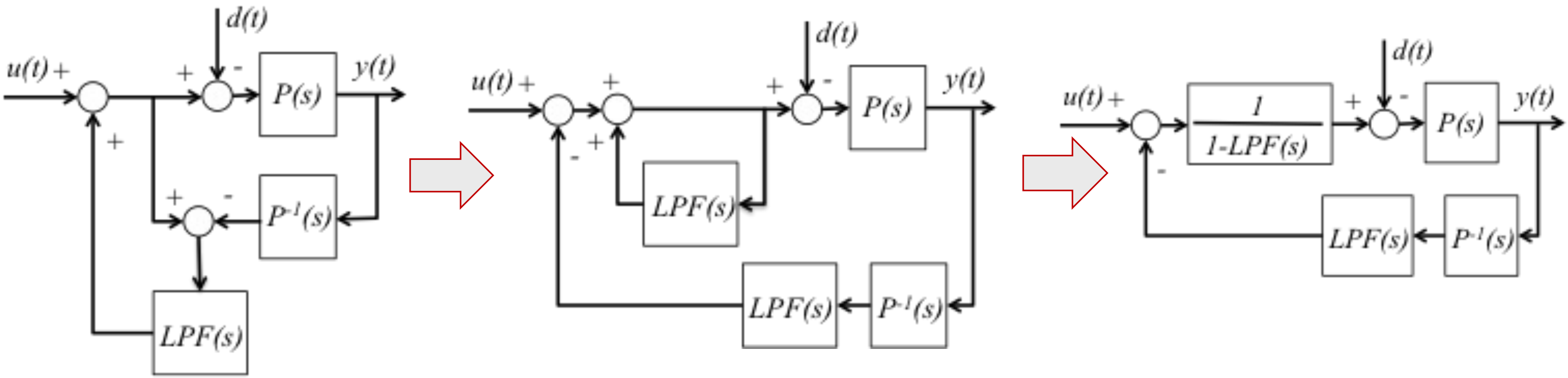
PID vs. DOB

- When designing the controller $C^*(s)$, the phase lag of the process $P(s)$ is 90° larger, due to the presence of the integrator in the controller.
  - This reduces the phase margin available and, in turn, limits the possibility in the design of $C^*(s)$.
  - As for the disturbance rejection, there is a full rejection of constant disturbances, while for other disturbances, this depends on $P(s)$ and $C^*(s)$ in a somewhat involved way

$$\frac{Y(s)}{D(s)} = \frac{sP(s)}{s + P(s)C^*(s)}$$

PID vs. DOB

- Let's now consider a simple implementation of the DOB, in which the actual process $P(s)$ corresponds to the nominal one (and its inverse, $P^{-1}(s)$, is used in the DOB)
  - The actual implementation of the DOB must take into account also a low-pass filter $LPF(s)$



PID vs. DOB

- Key results:

$$\frac{Y(s)}{U(s)} = \frac{\frac{1}{1-LPF(s)}P(s)}{1+\frac{LPF(s)}{P(s)}\frac{1}{1-LPF(s)}P(s)} = P(s)$$

$$\frac{Y(s)}{D(s)} = \frac{P(s)}{1+\frac{LPF(s)}{P(s)}\frac{1}{1-LPF(s)}P(s)} = P(s)(1-LPF(s))$$

  - the DOB does not alter the original process transfer function (and the phase profile of the system for which the controller must be designed)
  - the rejection is effective in the range of the bandwidth of $LPF(s)$, where $|1 - LPF(j\omega)| \approx 0$

PID vs. DOB

- The previous facts clearly set the difference between the integral-based and DOB-based disturbance rejection.
  - The first requires the inclusion of the integrator into the process to be controlled, with the reduction of the phase margin by 90°
  - The second does not alter the phase profile of the process to be controlled, thus resulting in an easier design, possibly with wider closed loop bandwidth.
  - Disturbance rejection performance with DOB is neatly stated by the design of the low pass filter, without complex relations with $C^*(s)$ and $P(s)$

PID vs. DOB

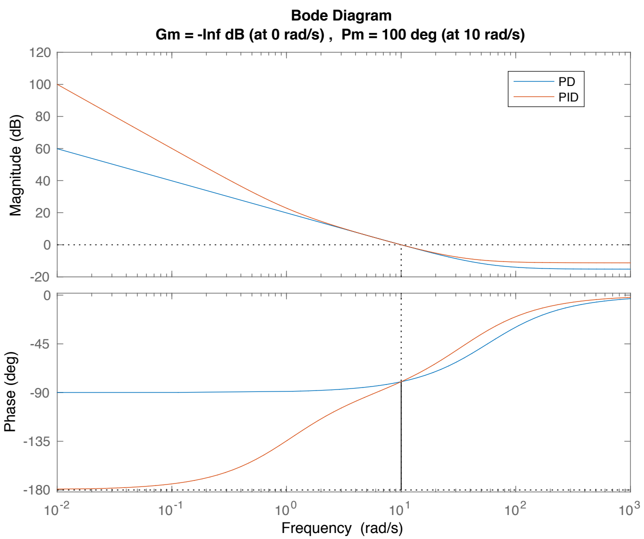
- In sum:
  - DOB and controllers with integral action may achieve the same result of getting rid of the effects of low frequency disturbances, but they are not equivalent.
  - Moreover, the compensation with DOB implements a kind of separation between the design of the controller and the disturbance compensator, leaving a greater freedom to the designer.
  - Let's see a simple performance comparison..

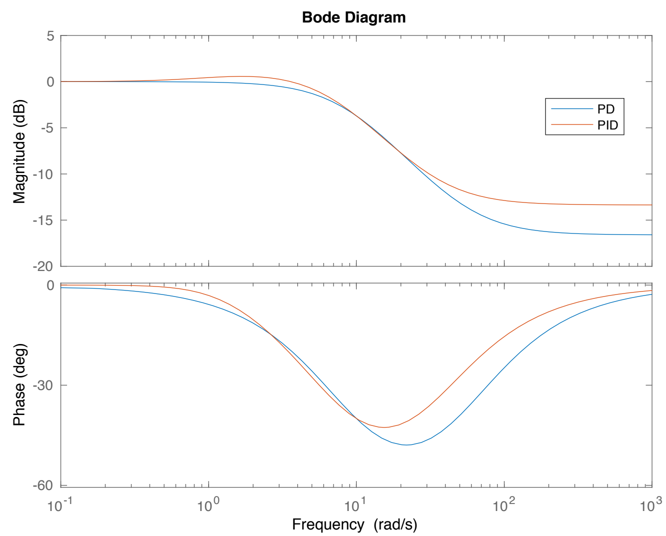
Disturbance rejection comparison

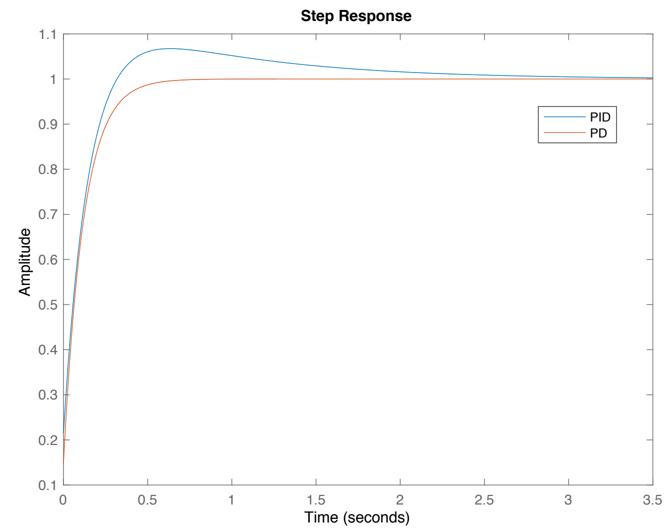
- To keep the problem simple, let's use the simplest dynamic model for a motor, $P(s) = 1/s$, i.e a motor in which torque constant $K_t = 1$ and $J = 1$
- Then, let's consider a pair of controllers, designed by following the standard Bode's method
  - The first is a PID and the second a PD, the latter to be used with the DOB-compensated motor.
  - Both controllers are designed for the same open loop crossing frequency $\omega_c=10\ rad/s$ and phase margin $\varphi=100°$
  - The low pass filter $LFP(s)$ has a bandwidth of $100\ rad/s$

Disturbance rejection comparison

– The two systems achieve similar performance in response to an input step, even if the presence of additional zeros in the PID controller (needed to compensate for phase delay caused by the integrator) leads (as expected) to a higher overshoot in the step response



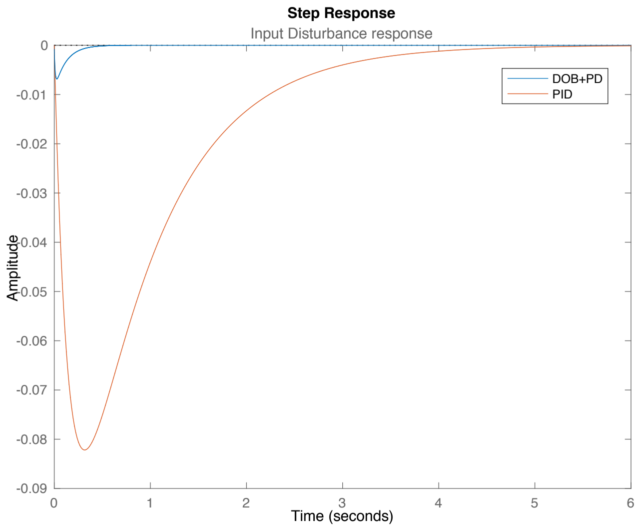


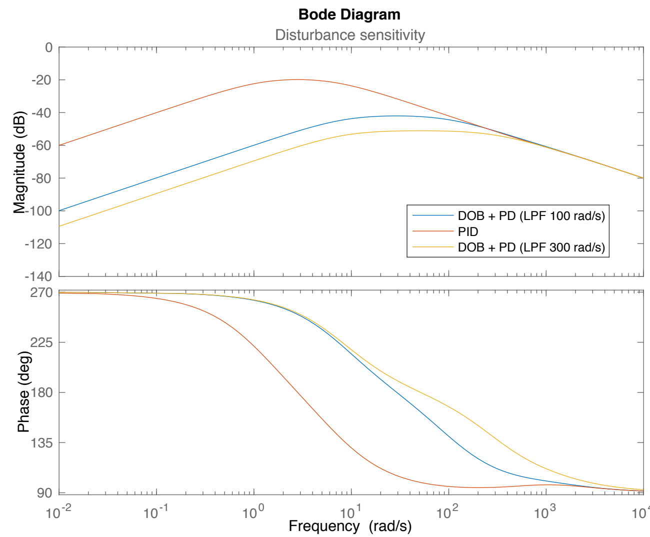


Disturbance rejection comparison

– The transient induced by the input disturbance is almost negligible with the DOB-based control, while the PID recovers a much larger effect in a much longer time.

– DOB-based control leads to a higher rejection, with a profile which is directly shaped by $LPF(s)$





Issues with DOB

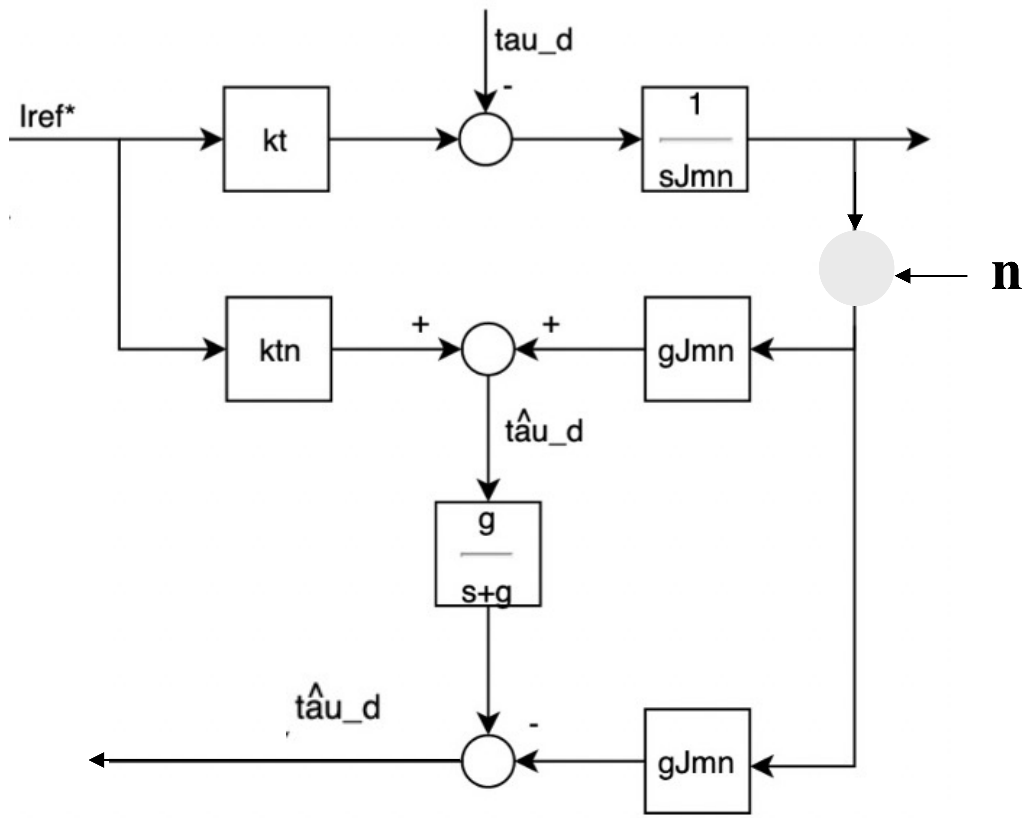
Issues with DOB

- DOB seems to magically solve many problems, but it has also some key issues, neglected at the beginning
  - How to properly choose the bandwidth of the filter applied to the measured velocity or position, in order to achieve the best trade-off between noise rejection and promptness?
  - How to define the best parameters of the ”nominal” process (in this case, the nominal motor torque constant K_{tn} and the nominal inertia J_n), while avoiding possible unstable behaviours?

Sensor noise effects in DOB

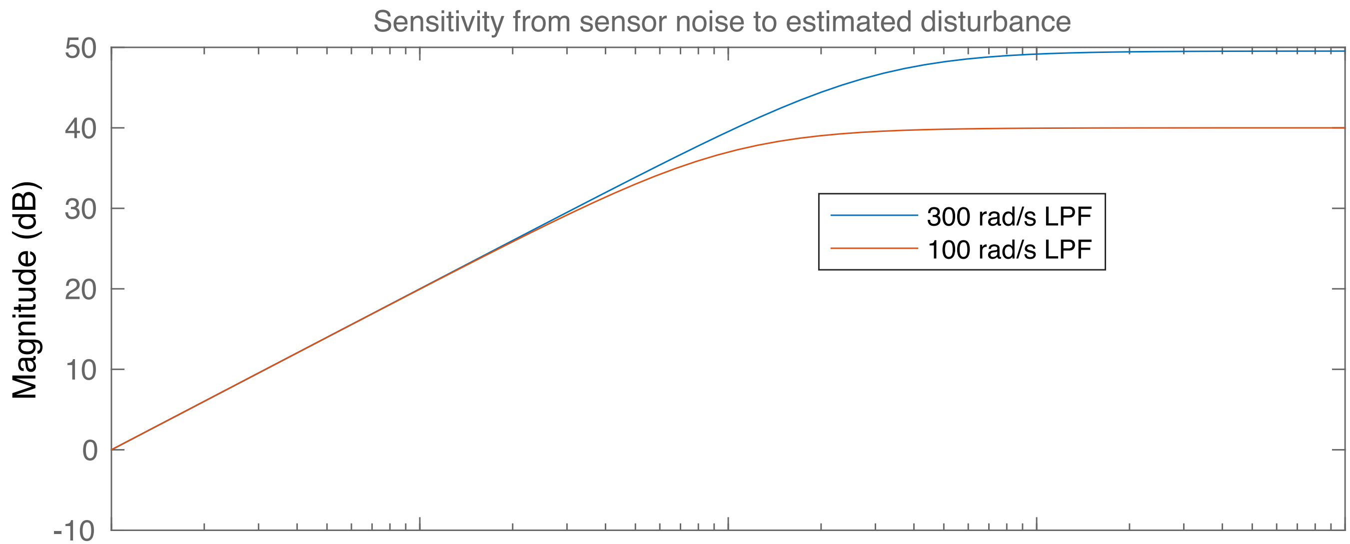
- The effectiveness of the DOB in disturbance rejection in strictly related to the bandwidth of the low pass filter LPF(s):
  - the higher the latter, the better the rejection
  - DOB designer may be led to choose the largest possible bandwidth for LPF(s)
- But the transfer function between sensor noise and estimated disturbance has a high frequency gain that increases with the bandwidth of LPF(s)

Sensor noise effects in DOB



Sensor noise effects in DOB

$$\frac{T_{dis}(s)}{N(s)} = -\frac{LPF(s)}{P(s)}$$



$$LPF(s) = \frac{g}{s+g}; \qquad P(s) = \frac{1}{s}; \quad \Rightarrow \frac{T_{dis}(s)}{N(s)} = \frac{-sg}{s+g}$$

Sensor noise effects in DOB

- Of course, it was possible to use a trial-and-error tuning
- This, however, is a typical scenario where an optimal filter design can be successfully applied
- In 1990, the DOB desing has been reformulated in a state-space framework, with an application to a flexible robotic joint
  - the disturbances on motor and robotic arm were represented as additional states of the system and the traditional approach to the disturbance observer was replaced by the design of a Kalman Filter

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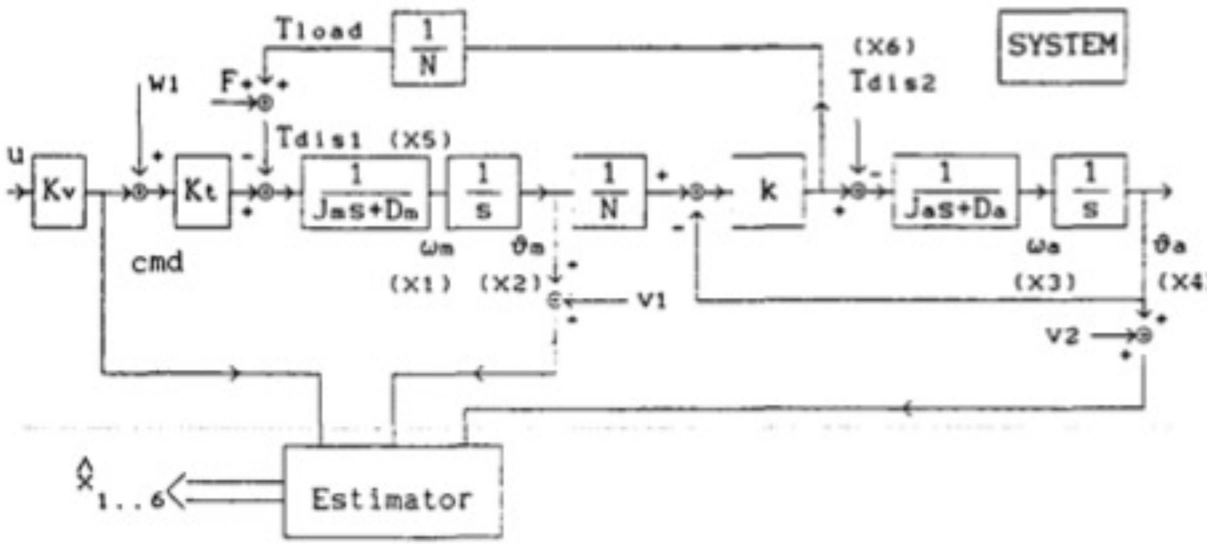
Sensor noise effects in DOB



K_t = torque constant;
K_v = current gain;
ϑ_m, ω_m = motor position and speed;
ϑ_a, ω_a = arm position and speed;
$T_{load}, T_{dis1}, T_{dis2}$ = load and disturbance torques;
F = static friction torque;

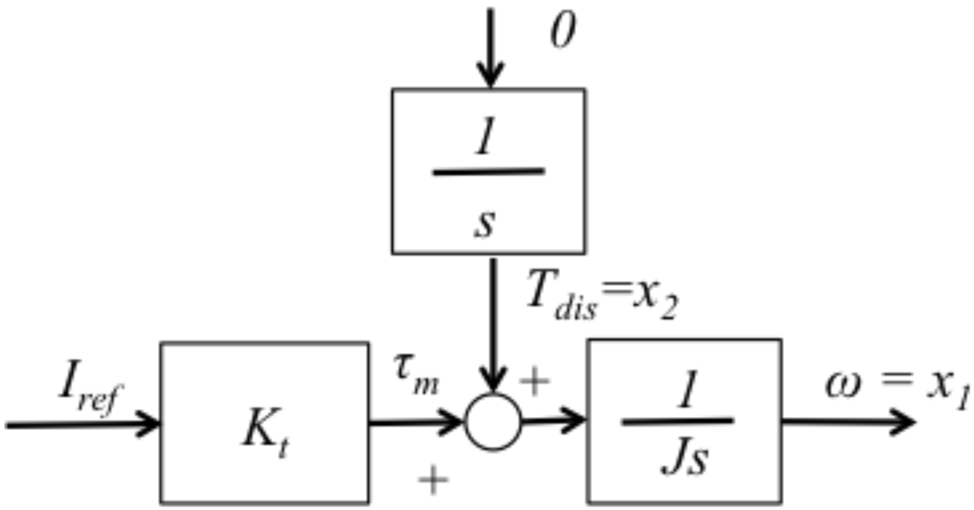
J_m, D_m = motor inertia and friction;
J_a, D_a = arm inertia and friction;
N = gear ratio;
k = gear box stiffness;
w_1 = system noise;
v_1, v_2 = measurement noise

DOB in state-space approach

- Of course, a major issue was to verify that the state-space approach was equivalent to that of the «traditional» DOB
  - The first step has been to find a state-space equivalent of the DOB concept, and this could be easily done by following the «zero order» modeling approach, where the unknown time-varying disturbance is conveniently represented with an additional state variable, having a zero-order model:

$$\dot{T}_{dis}(t) = 0$$

DOB in state-space approach



$$\mathbf{x}(t) = [\omega(t) \;\; T_{dis}]^T$$

$$u(t) = I_{ref}(t)$$

$$y(t) = \omega(t)$$

$$\dot{x}(t) = Ax(t) + Bu(t); y(t) = C_1 x(t); T_{dis}(t) = C_2 x(t)$$

$$A = \begin{bmatrix} 0 & 1/J \\ 0 & 0 \end{bmatrix}; B = \begin{bmatrix} K_t/J \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}; C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

DOB in state-space approach

- For such a system, an asymptotic state estimator can be as follows:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L[y(t) - C_1\hat{x}(t)]$$

- The estimated disturbance, then, is simply obtained as:

$$\hat{x}_2(t) = C_2\hat{x}(t)$$

DOB in state-space approach

- Disregarding the solution adopted for the computation of the estimator gain vector $L = [l_1\ l_2]^T$ (e.g. pole placement, Kalman Filtering etc.), the transfer function from system's input and output to the estimated disturbance results:

$$\hat{X}_2(s) = C_2[(sI - A + LC)^{-1}[BU(s) + LY(s)]] = [K_t U(s) - sJY(s)]\frac{-l_2}{Js^2 + Jl_1 s + l_2}$$

Nominal values…

- i.e. the estimate of the disturbance is the filtered version of the difference $K_tU(s)$-$sJY(s)$, as in the DOB!

DOB in state-space approach

- This simple result opened the possibility to use powerful tools, typical of the state-space framework (e.g. Kalman filtering), in the design of DOB.

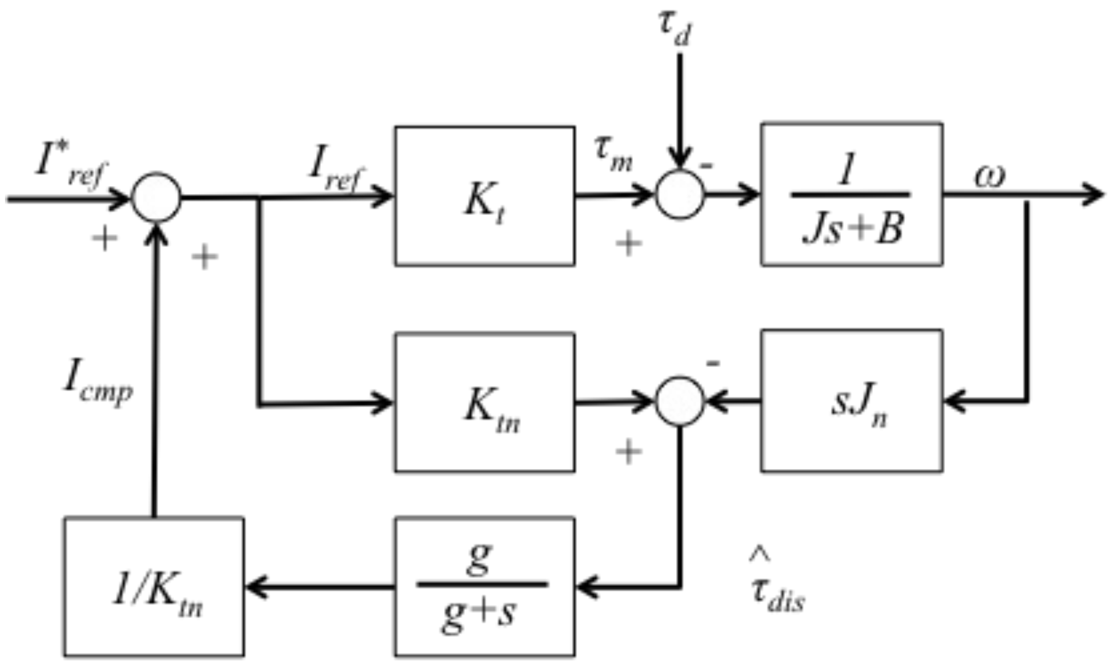
- In other words, it was the beginning of the end of the trial-and-error design of $LPF(s)$, for some time experienced in standard DOBs design.

Robustness of the DOB

- The most fascinating claim for the DOB was that the plant could be converted into a ”nominal” one, not affected by disturbances, at least in a certain frequency range.
  - Let’s see what really happens when the nominal parameters differ from the actual ones (which is clearly the standard condition)

Robustness of the DOB

- The DOB is designed considering the nominal plant parameters K_{tn} J_n, while the plant is characterized by K_t, J and B.



Robustness of the DOB

- If we compute the transfer function between the current reference and the velocity we get:

$$\frac{\Omega(s)}{I^*_{ref}(s)} = \frac{K_{tn}(s+g)}{J_n s(\frac{JK_{tn}}{J_n K_t}s + g + \frac{BK_{tn}}{J_n K_t})} = \frac{K_{tn}(s+g)}{J_n s(\alpha s + g')}$$

  – i.e. the DOB system behaves as the nominal one, in series to a pole-zero filter.

- Such filter may behave as a phase lead network, and this surely happens if

$$\alpha = \frac{JK_{tn}}{J_n K_t} \leq 1 \qquad B \geq 0$$

Robustness of the DOB

- Otherwise, the pole-zero pair introduces a phase delay, which may possibly lead to the instability of an outer control loop, designed on the nominal system
  - For such a reason, when considering a negligible friction (i.e. B ≈ 0) the basic guideline for the design of the nominal system was to choose a pair so that:

$$\frac{JK_{tn}}{J_n K_t} \leq 1$$

- This very simplistic approach to the problem of DOB robustness, was addressed in a formal and effective way by Umeno and Hori.

MEMS and DOB in Motion Control

MEMS & DOB

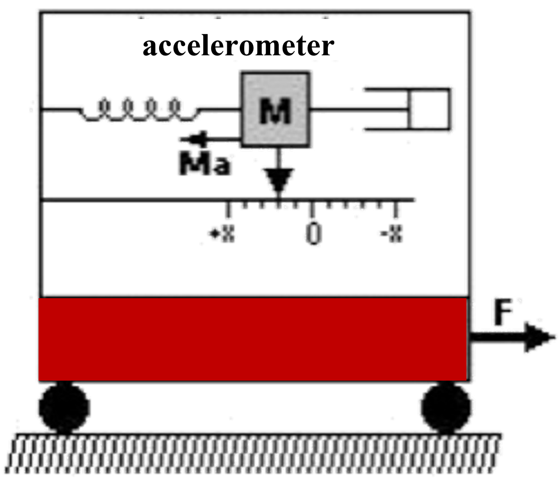
- Going back to the initial idea of DOB, we notice that we make use of the load acceleration, to get the equivalent disturbance

$$\tau_{dis} = K_{tn} I_{ref} - J_n \dot{\omega}$$

  - So, why not using a direct measurement of the acceleration, instead of resorting to noisy time derivative of the load position?
  - Solution: MEMS accelerometers
  - We will see a pair of applications in which the use of MEMS accelerometers brings a clear benefit

MEMS & DOB

- MEMS technology adapts the typical solutions for sensing acceleration to the micro-sized world
  - Accelerometers are still based on the measurement of acceleration-induced displacement of a seismic mass

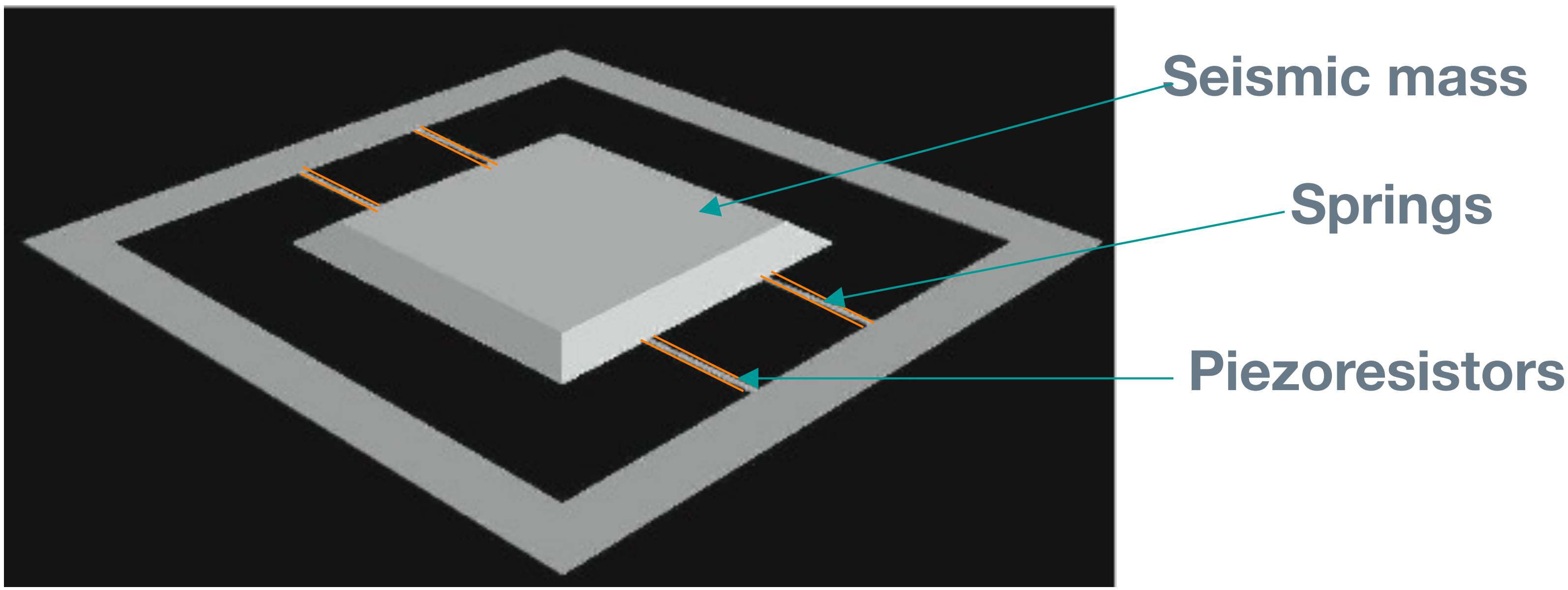


$$Ma = k \cdot \Delta x$$

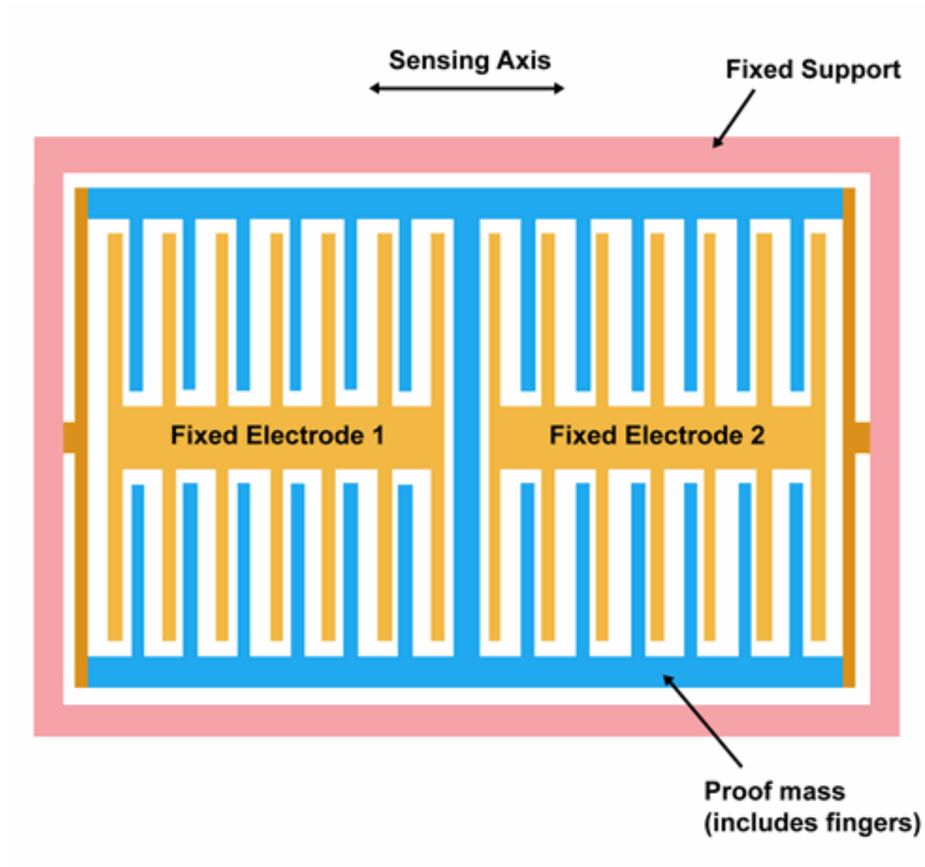
$$\Rightarrow a = \frac{k \cdot \Delta x}{M}$$

MEMS & DOB

- Sensing the deflection of a spring, supporting a seismic mass, is the key idea

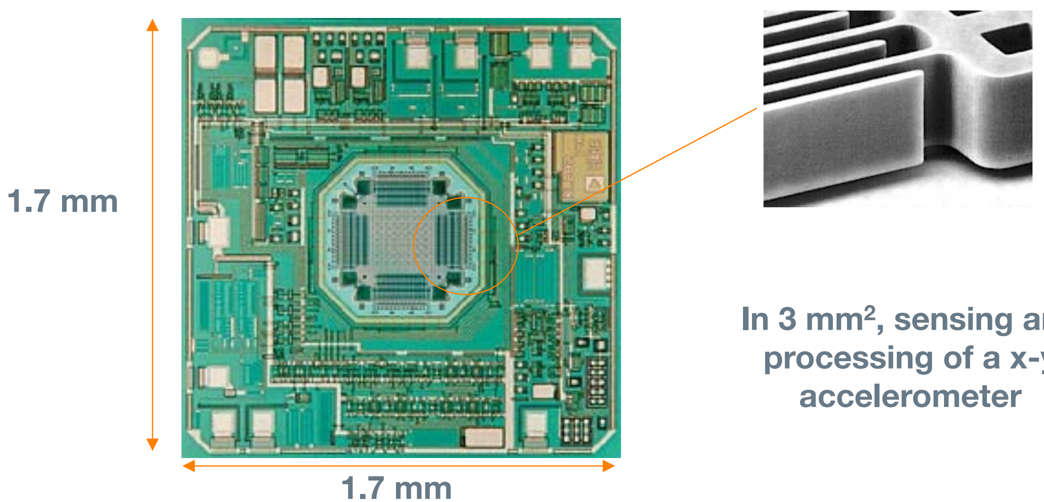


- Capacitive sensing is the most used solution

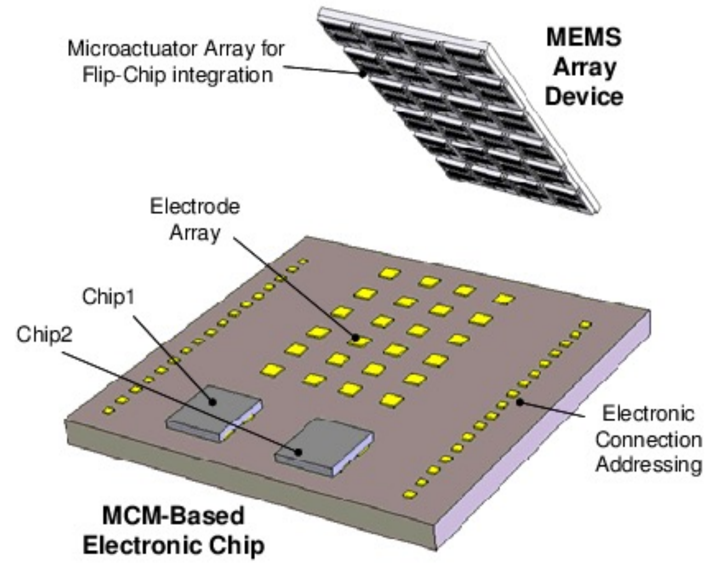


MEMS & DOB

- Sometimes, processing electronics and sensing are built on the same silicon chip
  - More often, sensing element and electronics are on different chips (to increase flexibility)



In 3 mm², sensing and processing of a x-y accelerometer



MEMS & DOB

The cost has been dropping while performances have been increasing:

- XYZ linear accelerometer for less than 1 USD, >1 kHz BW, 16 bits, etc..

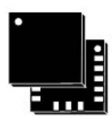
LIS3DHTR	Available at 4 distributors	ACTIVE	EAR99	NEC	Tape And Reel	LLGA 16 3x3x1.0	-	-	PHILIPPINES	0.792 / 1k

life.augmented

LIS3DH

MEMS digital output motion sensor: ultra-low-power high-performance 3-axis "nano" accelerometer

Datasheet - production data



LGA-16 (3x3x1 mm)

Features

- Wide supply voltage, 1.71 V to 3.6 V
- Independent IO supply (1.8 V) and supply voltage compatible
- Ultra-low-power mode consumption down to 2 µA
- ±2*g*/±4*g*/±8*g*/±16*g* dynamically selectable full scale
- I^2C/SPI digital output interface
- 16-bit data output

- Display orientation
- Gaming and virtual reality input devices
- Impact recognition and logging
- Vibration monitoring and compensation

Description

The LIS3DH is an ultra-low-power high-performance three-axis linear accelerometer belonging to the "nano" family, with digital I^2C/SPI serial interface standard output. The device features ultra-low-power operational modes that allow advanced power saving and smart embedded functions.

The LIS3DH has dynamically user-selectable full scales of ±2*g*/±4*g*/±8*g*/±16*g* and is capable of measuring accelerations with output data rates from 1 Hz to 5.3 kHz. The self-test capability allows the user to check the functioning of the sensor in the final application. The device may be

3x3x1 mm package

0.792 USD

5.3 kHz BW

Low resolution position sensors in DOB-based motion control systems

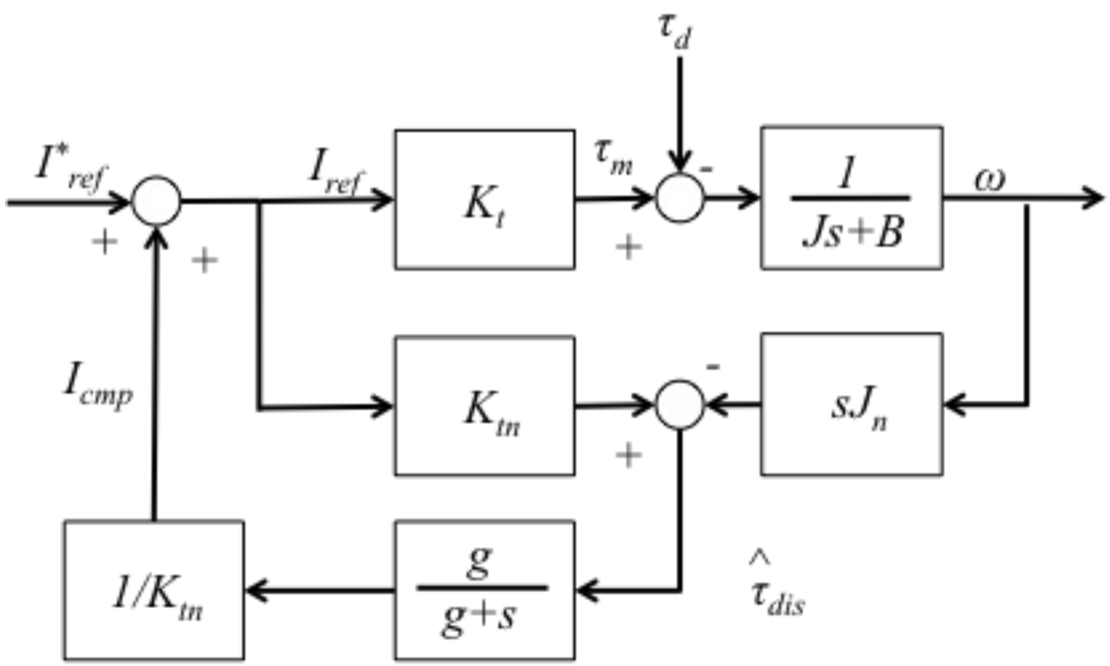
Positioning Performance Improvement with MEMS sensors

- Old and low cost motion control systems often rely on low resolution positions sensors
    - Low positioning accuracy
    - Limited achievable speed, due to high noise in speed, obtained by differentiation of the position measurement
- Replacing the existing low-resolution position sensors with higher resolution ones may require the complete redesign of the system, as the new sensor may not fit in the available space of the existing plant

Positioning Performance Improvement with MEMS sensors

- Otherwise, it is possible to use sophisticated hardware for encoder signal processing
  - Smart time-stamping and selection of encoder events is used to virtually increase the resolution in position measurement
- A common practice in motion control systems is also to make use of Disturbance Observers (DOB), which rely on the availability of an estimate of the acceleration of the mechanical load to be controlled
  - Using load position and its derivatives severely limits the DOB performance when using low-resolution position sensors

DOB


τ_d
I^*_{ref}
I_{ref}
K_t
τ_m
$\frac{1}{Js+B}$
ω
I_{cmp}
K_{tn}
sJ_n
$1/K_{tn}$
$\frac{g}{g+s}$
$\hat{\tau}_{dis}$

Positioning Performance Improvement with MEMS sensors

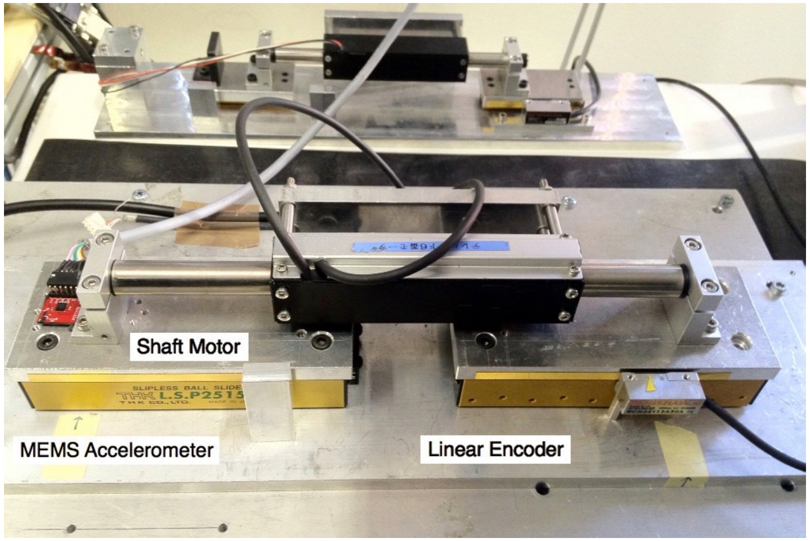
- We propose the use of a Kalman Filter (KF), which implements a sensor fusion, in order to reduce the effects of the quantization noise affecting the measured position.
- In particular, the KF utilizes the measurements provided by a low-cost MEMS accelerometer to enhance the quality of position and velocity estimates, to be used in the closed loop position control of a positioning system.
- KFs + accelerometers have been used by others: what is the difference here?

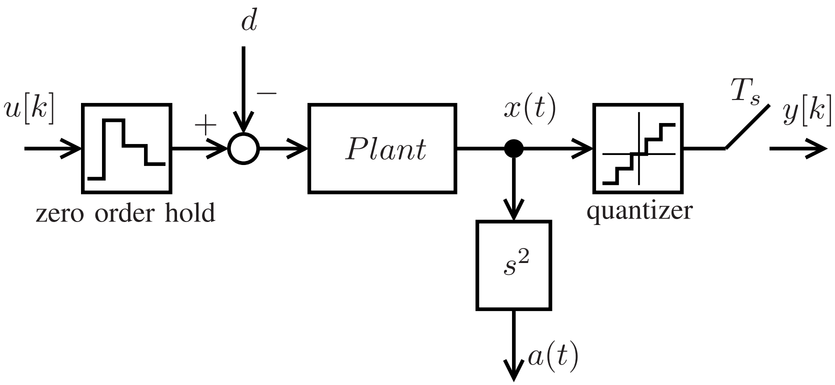
Positioning Performance Improvement with MEMS sensors

- We combined some relevant features:
  - Kinematic KF: not relying on the actual plant dynamics, the estimates are insensitive to plant variations (robustness)
  - KF estimates of position and velocity are used in place of the measurements and their derivatives in the outer servo loops
  - Augmented model for acceleration measurement
    - Bias and drift on measurements are estimated and compensated
  - KF is systematically tuned on the actual plant, by using whiteness tests on estimation error
    - Optimally tuned KF produced smooth estimates of kinematic variables and disturbances in a wider frequency range, compared to standard DOB implementations, based on position measurements

Kalman estimator utilizing a MEMS accelerometer

- We developed the model and the Kalman estimator for a rigid, single degree of freedom servo positioning system:
  - a linear motor on which a position sensor and a MEMS accelerometer provide the load mass position and acceleration, respectively.



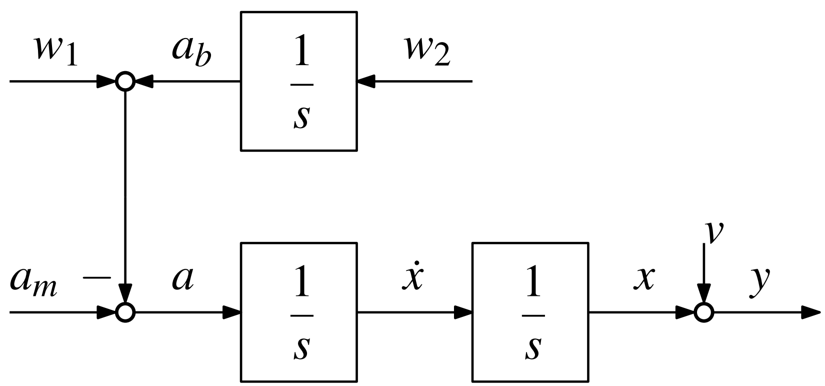


Kalman estimator utilizing a MEMS accelerometer

- When considering the bias and noise affecting the acceleration measurement, an augmented model of the plant must be considered
  - The actual acceleration a as the sum of the measured one (a_m) plus a random walk-like bias (a_b) and a noise, i.e.

$$a = a_m + a_b + w_1$$

$$\dot{a}_b = w_2$$



- The definitions can be combined in the following space state model, describing the dynamics between the acceleration measurement a_m and the measured position y:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B_1} a_m + \boldsymbol{B_2}\boldsymbol{w} \\ y &= \boldsymbol{C}\boldsymbol{x} + v \end{aligned}$$

where $\boldsymbol{x} = [x,\ \dot{x},\ a_b]^T$, $\boldsymbol{w} = [w_1,\ w_2]^T$ and

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}, \boldsymbol{B_1} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \boldsymbol{B_2} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

Kalman estimator utilizing a MEMS accelerometer

- A time varying KF can be easily implemented for the discrete time version of the plant:

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{\Phi}\boldsymbol{x}_k + \boldsymbol{\Gamma} a_{m,k} + \boldsymbol{G}\boldsymbol{w}_k \\ y_k &= \boldsymbol{H}\boldsymbol{x}_k + v_k \end{aligned}$$

$$\boldsymbol{\Phi} = e^{\boldsymbol{A}T_s}, \qquad \boldsymbol{\Gamma} = \int_0^{T_s} e^{\boldsymbol{A}\tau}\boldsymbol{B_1}d\tau,$$

$$\boldsymbol{G} = \int_0^{T_s} e^{\boldsymbol{A}\tau}\boldsymbol{B_2}d\tau, \qquad \boldsymbol{H} = \boldsymbol{C}$$

$$\boldsymbol{x} = [x,\ \dot{x},\ a_b]^T.$$

<u>estimation:</u>

$$\begin{aligned} \boldsymbol{P}_{k,k-1} &= \boldsymbol{\Phi}\boldsymbol{P}_{k-1,k-1}\boldsymbol{\Phi}^T + \boldsymbol{Q} \\ \hat{\boldsymbol{x}}_{k,k-1} &= \boldsymbol{\Phi}\hat{\boldsymbol{x}}_{k-1,k-1} + \boldsymbol{\Gamma} a_{m,k} \end{aligned}$$

<u>update:</u>

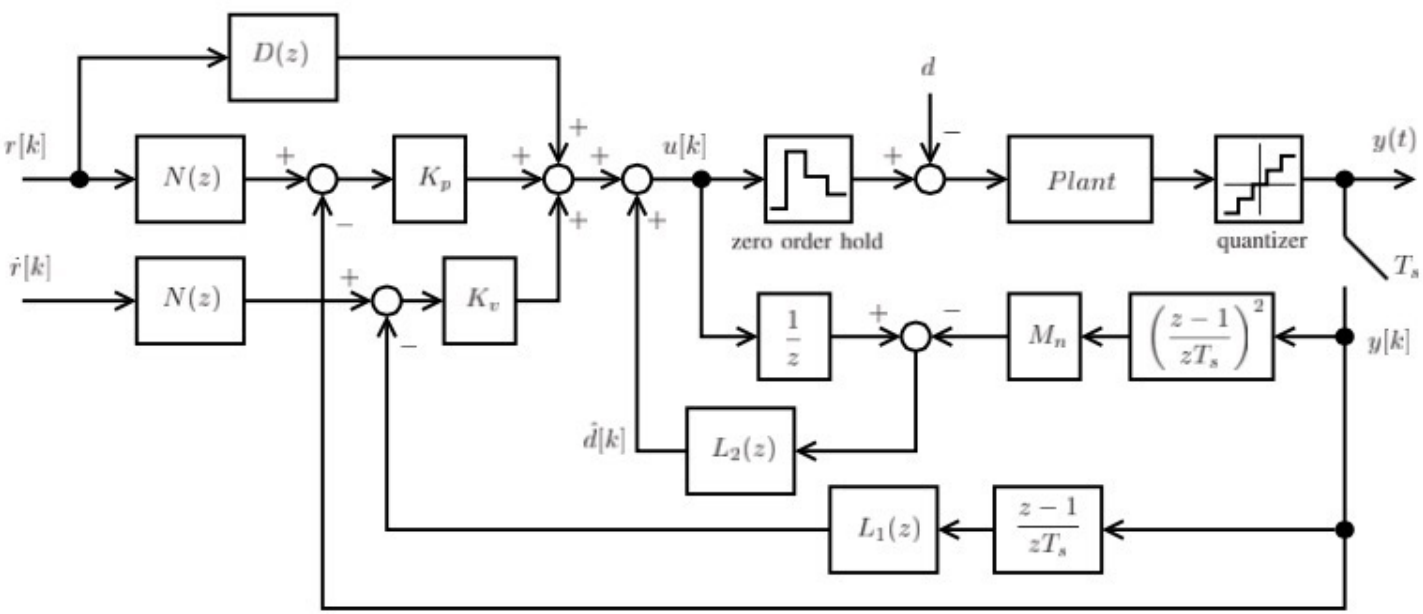
$$\begin{aligned} \boldsymbol{K}_k &= \boldsymbol{P}_{k,k-1}\boldsymbol{H}^T(\boldsymbol{H}\boldsymbol{P}_{k,k-1}\boldsymbol{H}^T + \sigma_R^2)^{-1} \\ \hat{\boldsymbol{x}}_{k,k} &= (\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H})\,\hat{\boldsymbol{x}}_{k,k-1} + \boldsymbol{K}_k y_k \\ \boldsymbol{P}_{k,k} &= (\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H})\,\boldsymbol{P}_{k,k-1} \end{aligned}$$

P-DOB vs. A-DOB

- Standard implementation of the DOB makes use of filtered version of the double derivative of the position to obtain an estimate of the acceleration and disturbance
    - P-DOB
- Additionally, load velocity is obtained by filtered differentiation of the position.
- Such filters are usually experimentally tuned, in order to obtain a good compromise between overall achieved BW and residual effects of quantization noise

P-DOB vs. A-DOB

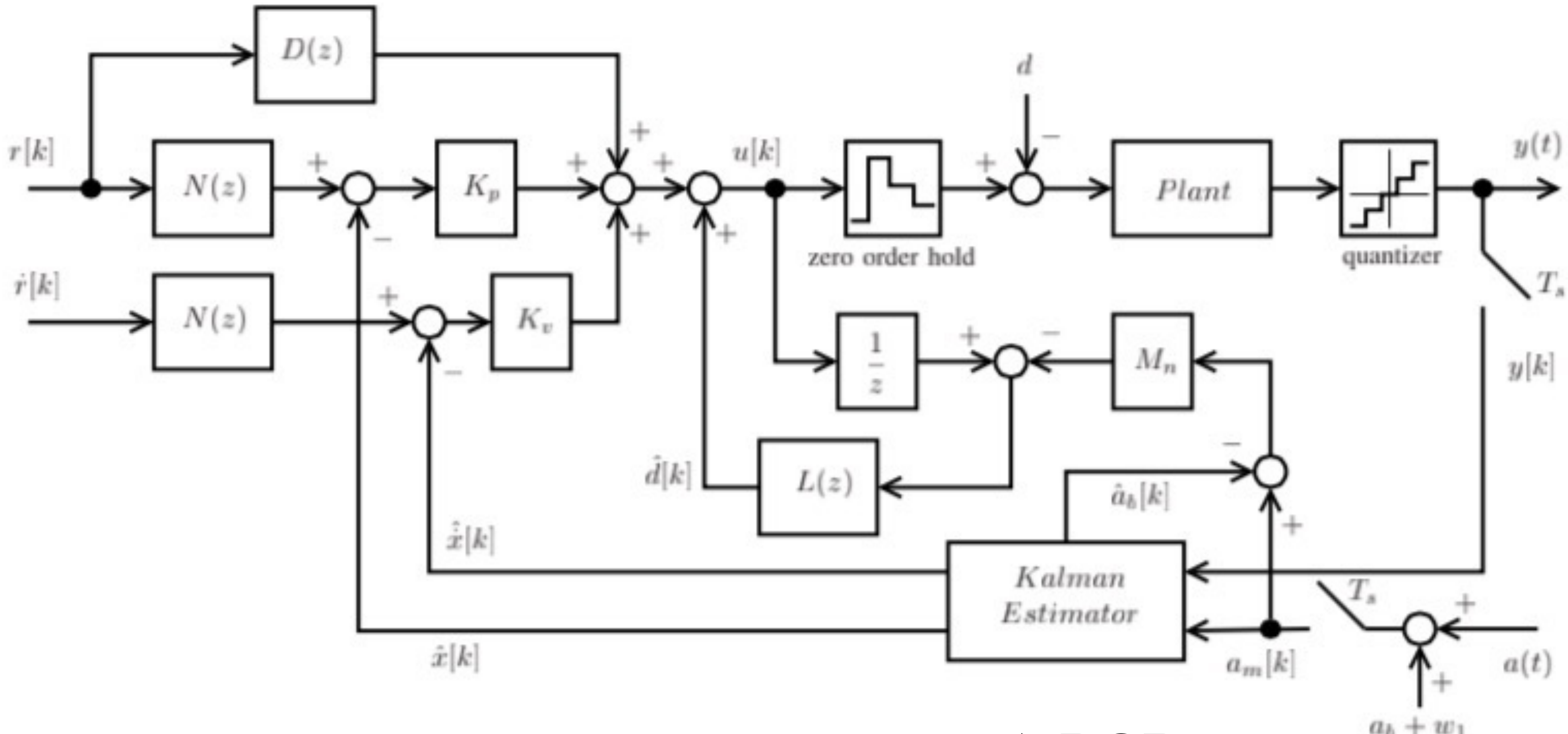
- We compared the results obtained with p-DOB based position servo with those obtained with acceleration-based DOB (A-DOB)



P-DOB

P-DOB vs. A-DOB

- Position and velocity controllers are 2-DOF

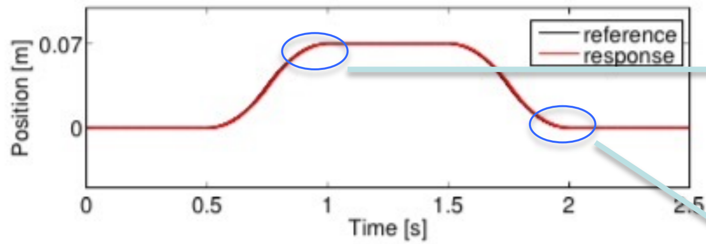


A-DOB

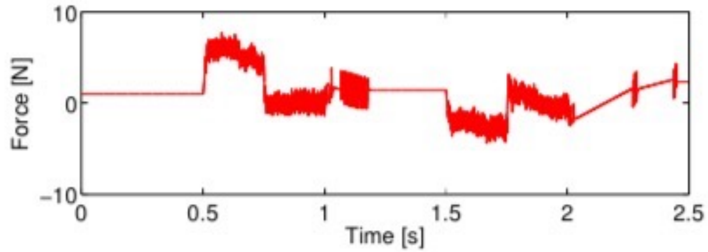
P-DOB vs. A-DOB

- The main differences are in the estimation of disturbance and kinematic variables.
- Not requiring a narrow filtering of the estimates, a wider BW and faster convergence can be achieved with the same position sensors.
- Actuator force results smoother with A-DOB
- Robustness against load variations is increased with A-DOB, as it uses a kinematic KF.

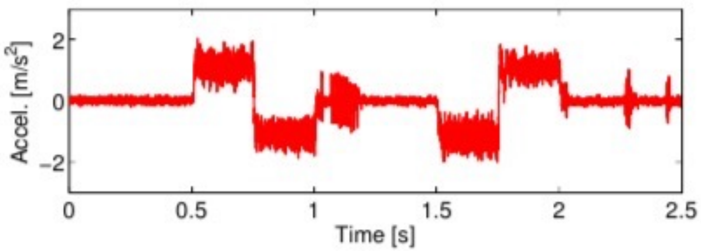
P-DOB vs. A-DOB



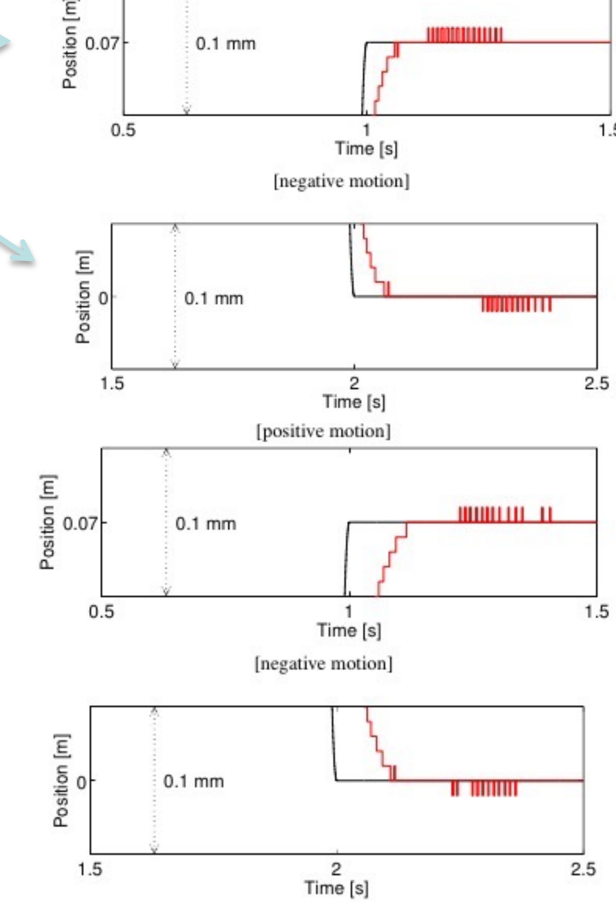
(a) table position



(b) thrust force



(c) acceleration

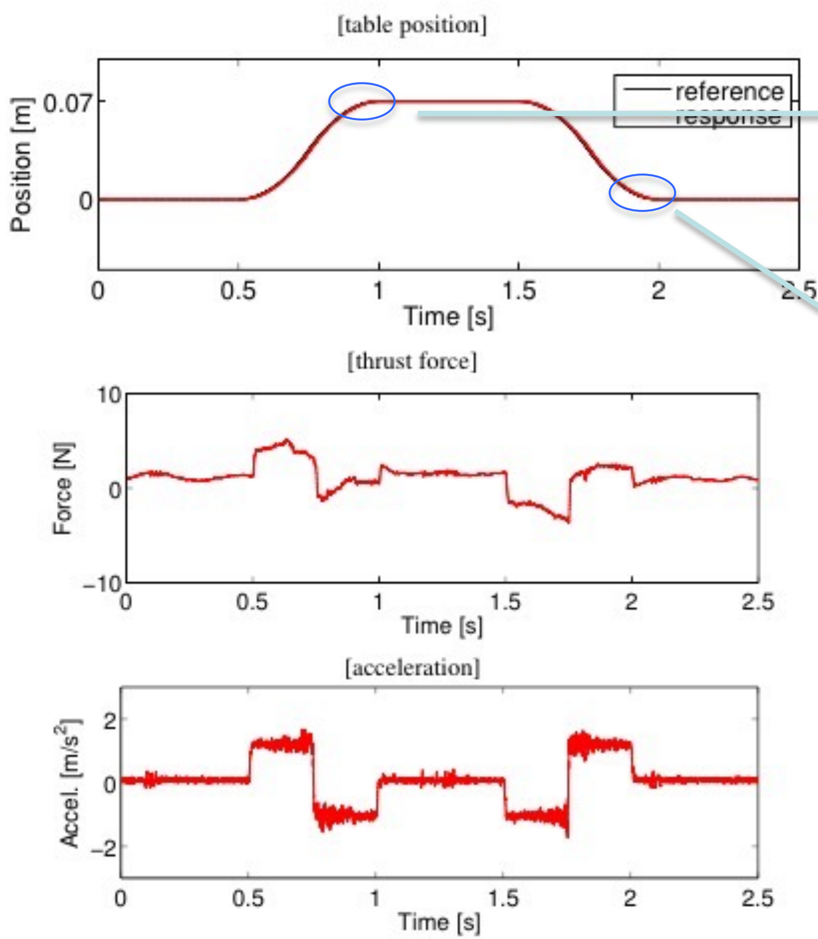


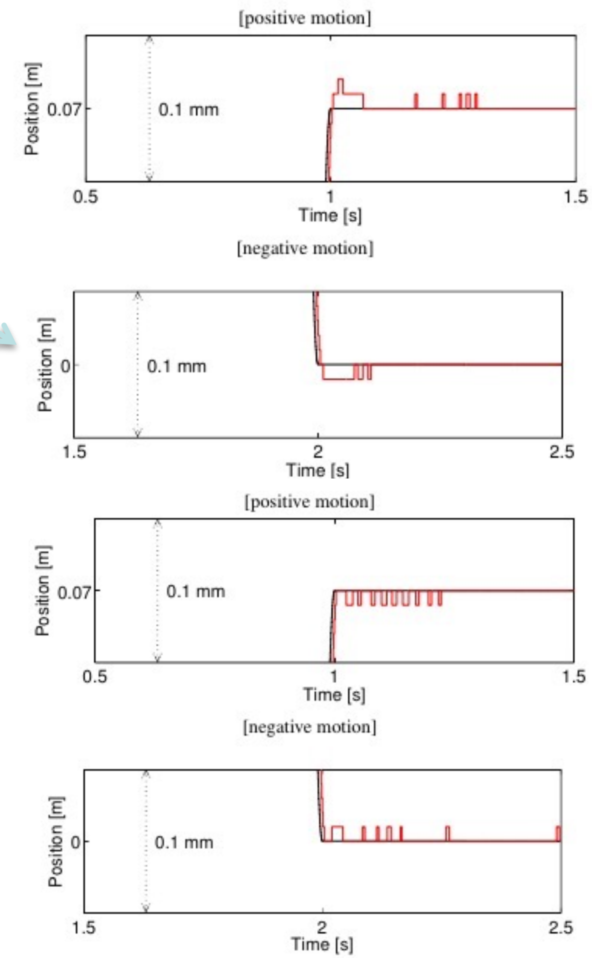
Mn

2Mn

P-DOB

P-DOB vs. A-DOB





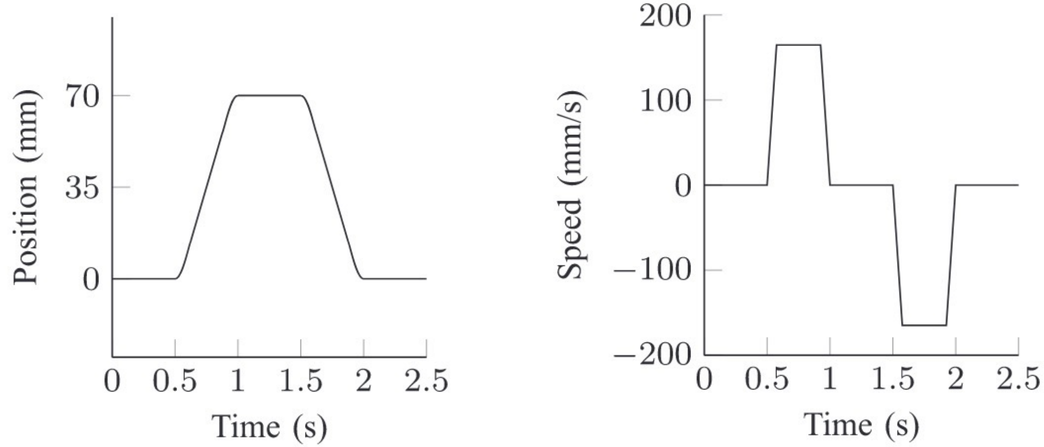
Mn

2Mn

A-DOB

Bias/drift in MEMS measurements

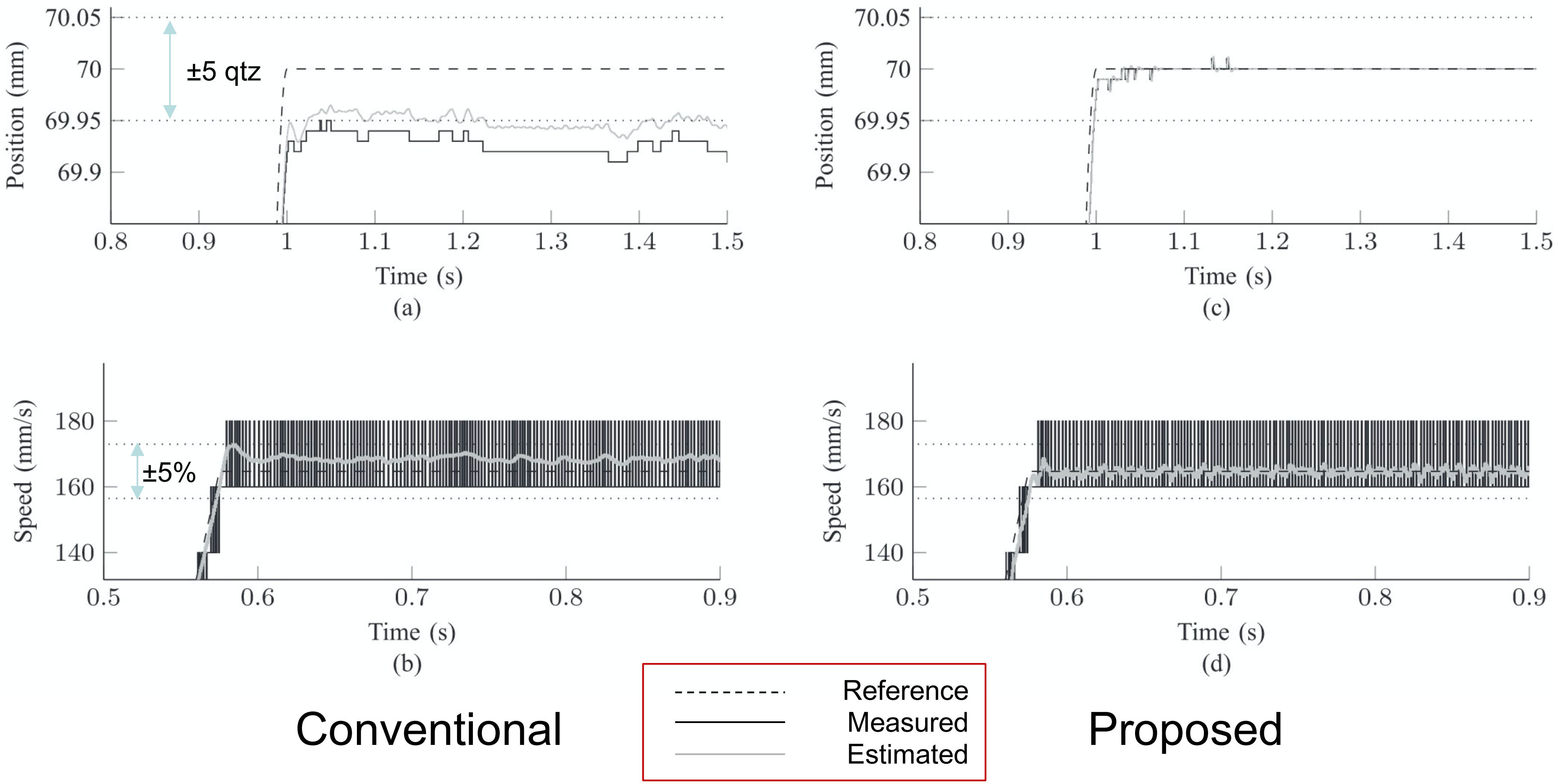
- Bias could be removed at start-up, but this won't be enough, because zero-g output tends to drift with time and temperature.
  - The proposed sensor fusion takes advantage of the availability of a coarse position measurement to update the value of a_b
  - With a constant bias compensation, the control may show an error in both positioning and velocity accuracy



(a) (b)

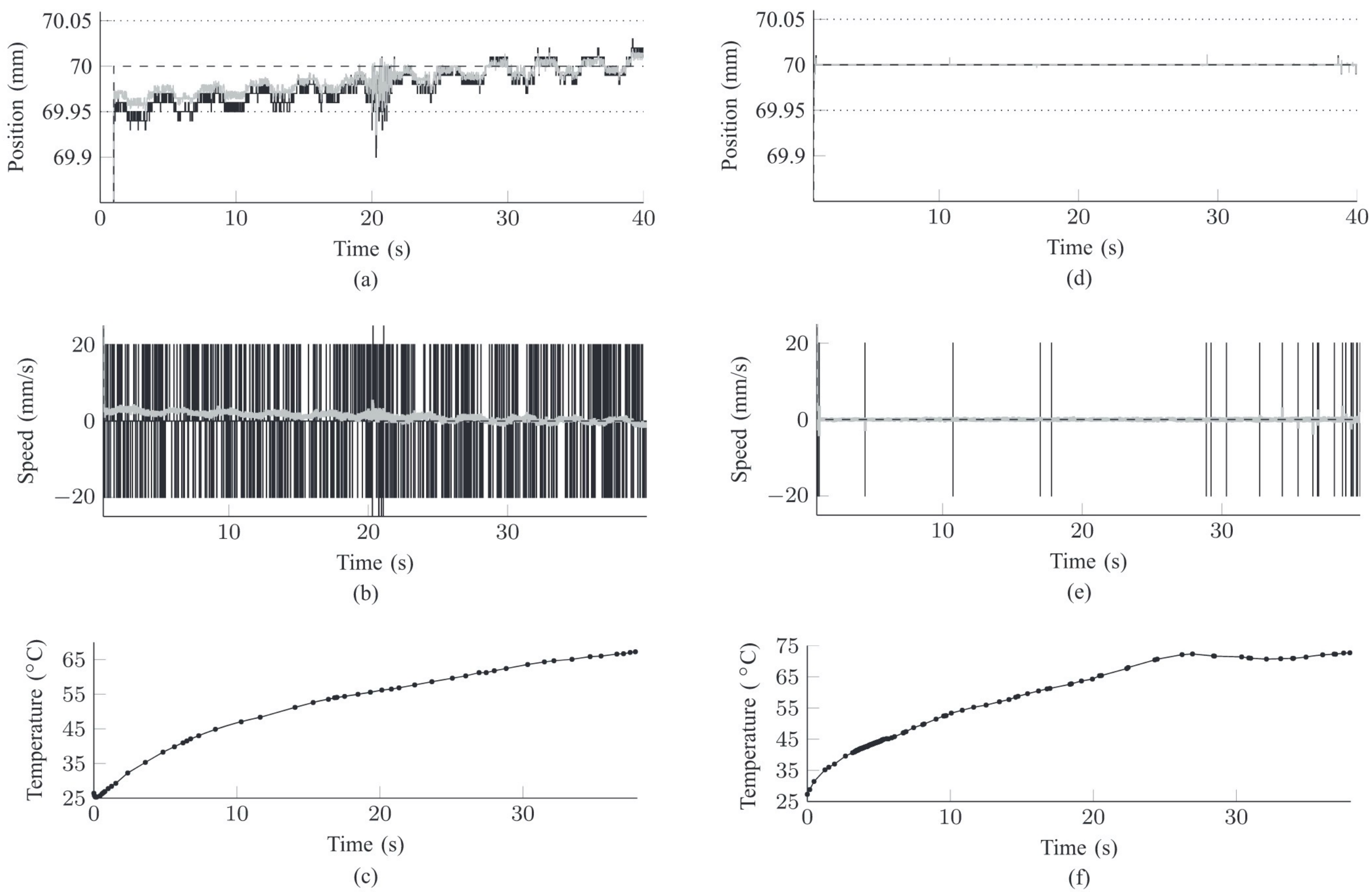
Bias/drift in MEMS measurements

- Unproperly compensated bias leads to errors:



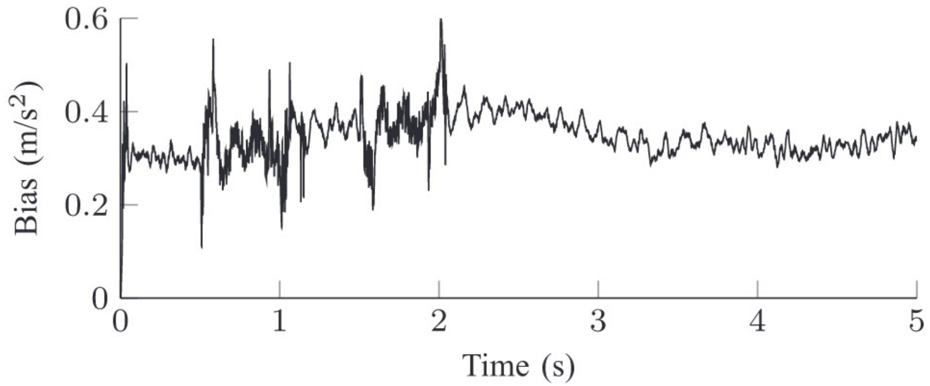
Bias/drift in MEMS measurements

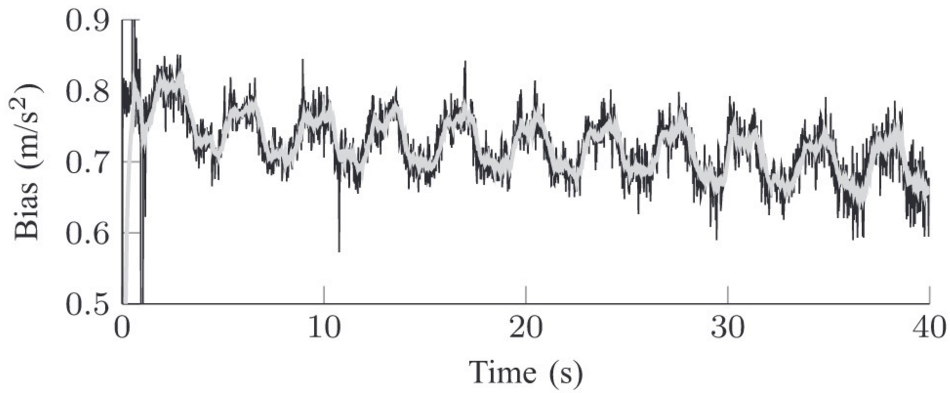
- Most of the drift is caused by temperature variations



Estimated drift

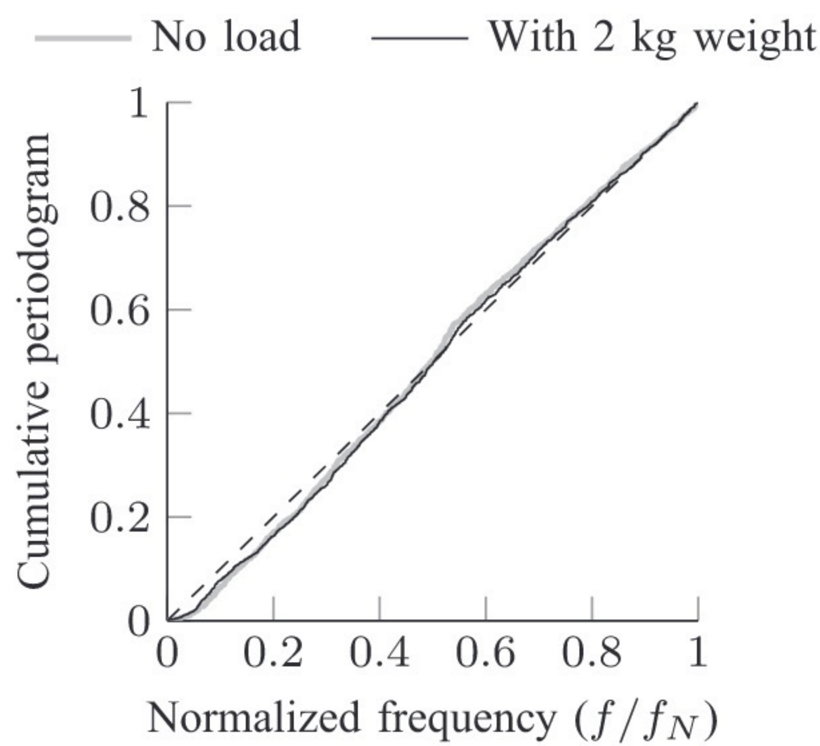
- It may change with load position, time, temperature





KF tuning

- A crucial issue is the tuning of the KF, i.e. the proper choice of the measurement and model noise.
- Encoder and accelerometer noise variances can be obtained experimentally
- Accelerometer random walk is experimentally tuned with a whiteness test (Bartlett cumulated periodogram)



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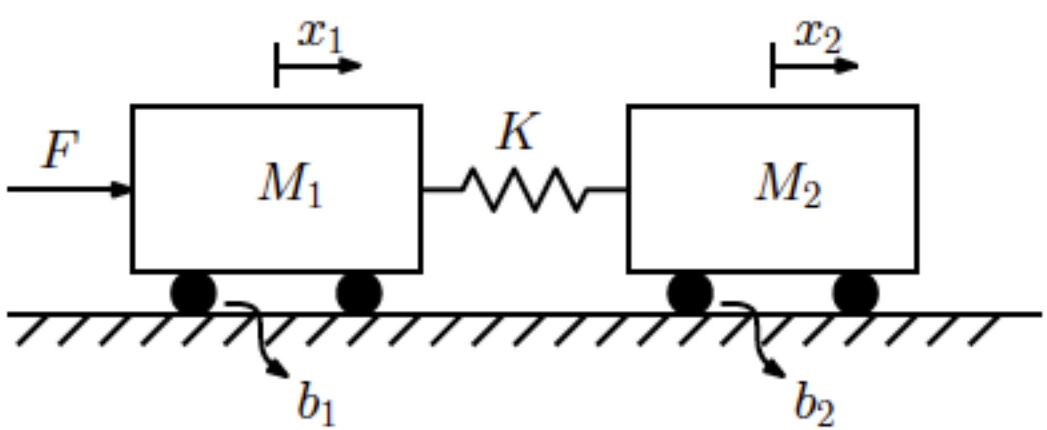
Use of Loadside MEMS Accelerometers in Servo Positioning of Two–Mass–Spring Systems

Control of two–mass–spring systems

- The previous example was applied to a rigid servo-positioner.
- In many industrial applications, servo–positioning devices are composed of an electric motor, connected to the mechanical load, through elastic elements



Control of two–mass–spring systems

- The problem of two–mass resonant systems has been studied for long time, and the main issue targeted has been the development of accurate and robust servo positioning devices, without making use of a load–side position sensor.
- By using properly designed observers, it is possible to obtain accurate information on the load position, but the robustness of control laws based on this solution is weak against plant parameter variations (e.g. stiffness, friction, inertia)

Control of two–mass–spring systems

- Robust controllers for uncertain two–mass–spring systems, using different approaches (e.g., μ–analysis, H∞, LMI , non–linear observers etc.) are usually designed in a conservative way and they do not provide a level of performance close to that achievable with a load–side position sensor.
- The damping of the oscillations between actuator and load mass can be easily achieved in case of availability of load side position and/or velocity
- When an estimator is used, if the model and the actual system are not perfectly matched, the active damping is not effective, or it even worsens the performance.

USE of load side MEMS accelerometers

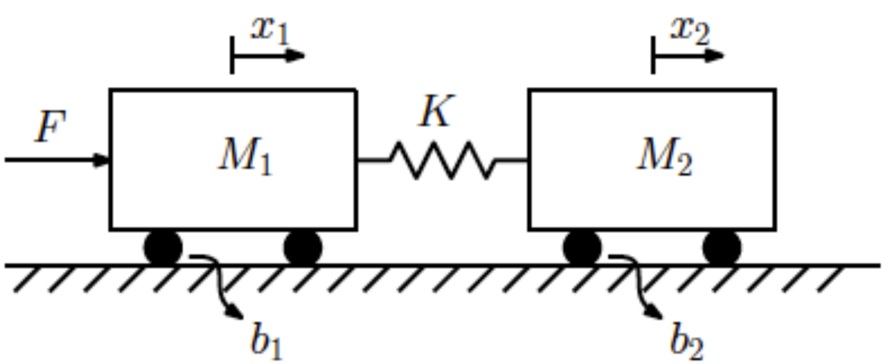
- We show here the feasibility, the performance and the robustness of an optimal state feedback control, in which the full state of the system is obtained by a Kalman Filter
  - The main idea is to make use of a load side, low-cost MEMS accelerometer to implement a robust estimate of the load position and velocity
- The use of accelerometers in position control requires some extra care, as in the previous case:
  - The presence of unknown biases and drifts in acceleration measurements, leads to diverging estimates of the load velocity
  - An augmented model of the acceleration measurement has been properly embedded in the model of a two–mass–spring system
- It will be shown that the proposed solution recovers the robustness of a full state optimal feedback (LQR)

System model

- The standard space state model is SISO and considers a force input and the co-located position measurement

$$\dot{\mathbf{x}} = A_c \mathbf{x} + B_c u + B_{cw} w$$

$$y = C_c \mathbf{x} + v$$



$$\mathbf{x} = [\dot{x}_1\ x_1\ \dot{x}_2\ x_2]^T\ ,\ u = F\ ,\ y = x_1$$

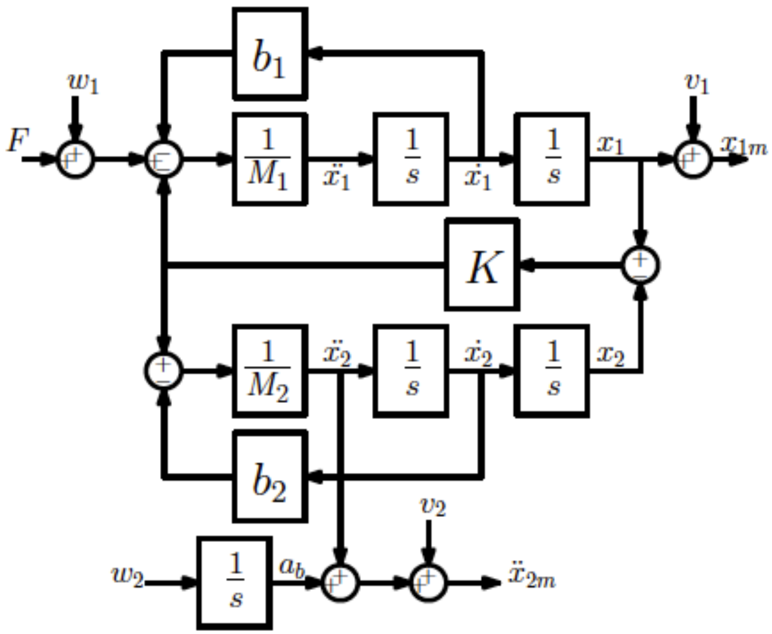
$$A_c = \begin{bmatrix} -\frac{b_1}{M_1} & -\frac{K}{M_1} & 0 & \frac{K}{M_1} \\ 1 & 0 & 0 & 0 \\ 0 & \frac{K}{M_2} & -\frac{b_2}{M_2} & -\frac{K}{M_2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_c = \begin{bmatrix} \frac{1}{M_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad B_{cw} = \begin{bmatrix} \frac{1}{M_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_c = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

Two–mass–spring linear system with acceleration measurement

- Considering a load–side acceleration measurement, comprising (again) a zero–order stochastic model (i.e. a random walk) of the bias plus drift a_b, the model is augmented by a state variable:



$$\ddot{x}_{2m} = \ddot{x}_2 + a_b + v_2$$

$$\dot{a}_b = w_2$$

Two–mass–spring linear system with acceleration measurement

- Considering a load–side acceleration measurement, comprising (again) a zero–order stochastic model (i.e. a random walk) of the bias plus drift a_b, the model is augmented by a state variable:

$$\mathbf{x} = [\dot{x}_1 \; x_1 \; \dot{x}_2 \; x_2 \; a_b]^T \;,\; u = F \;,\; y = [x_{1m} \; \ddot{x}_{2m}]^T$$

$$\mathbf{A_c} = \begin{bmatrix} -\frac{b_1}{M_1} & -\frac{K}{M_1} & 0 & \frac{K}{M_1} & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{K}{M_2} & -\frac{b_2}{M_2} & -\frac{K}{M_2} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \;,\; \mathbf{B_c} = \begin{bmatrix} \frac{1}{M_1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} ,$$

$$\mathbf{B_{cw}} = \begin{bmatrix} \frac{1}{M_1} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \;, \mathbf{C_c} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{b_1}{M_1} & -\frac{K}{M_1} & -\frac{b_2}{M_1} & \frac{K}{M_1} & 1 \end{bmatrix}$$

$$\dot{\mathbf{x}} = \mathbf{A_c x} + \mathbf{B_c} u + \mathbf{B_{cw} w},$$
$$\mathbf{y} = \mathbf{C_c x} + \mathbf{v}$$

$$\mathbf{Q_c} = \mathbf{E}[\mathbf{w}\mathbf{w}^T] \;,\; \mathbf{R_c} = \mathbf{E}[\mathbf{v}\mathbf{v}^T]$$

LQG and LQR control

- The control of a two–mass–spring system can be performed by using several techniques.
- We have implemented an optimal state–feedback control, in which the state is either fully available (LQR) or estimated by a KF (LQG), both built around the discrete time version of the state space model.

$$\mathbf{x}_{k+1} = \mathbf{A}_d \mathbf{x}_k + \mathbf{B}_d u_k + \mathbf{B}_{dw} \mathbf{w}_k$$
$$\mathbf{y}_k = \mathbf{C}_d \mathbf{x}_k + \mathbf{v}_k$$

$$\mathbf{A}_d = e^{\mathbf{A}_c T_s} \mathbf{B}_d = \int_0^{T_s} e^{\mathbf{A}_c \tau} \mathbf{B}_c d\tau$$

$$\mathbf{B}_{dw} = \int_0^{T_s} e^{\mathbf{A}_c \tau} \mathbf{B}_{cw} d\tau \ , \ \mathbf{C}_d = \mathbf{C}_c$$

$$\mathbf{Q}_d = \int_0^{T_s} e^{\mathbf{A}_c \tau} \mathbf{Q}_c e^{\mathbf{A}_c{}^T \tau} d\tau \ , \ \mathbf{R}_d = \frac{\mathbf{R}_c}{T_s}$$

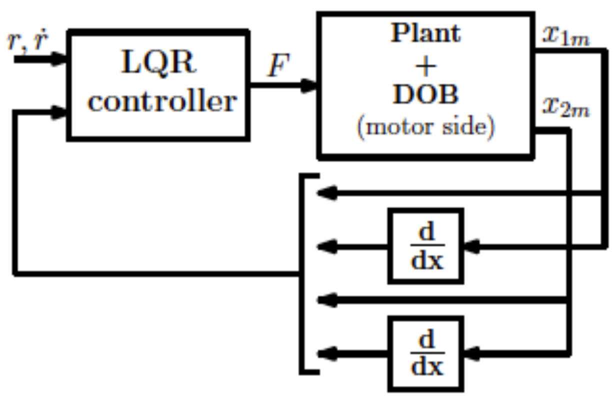
- LQR is known for its excellent robustness, which is usually lost in LQG with standard KF as state estimator.

Standard and acceleration-aided KF

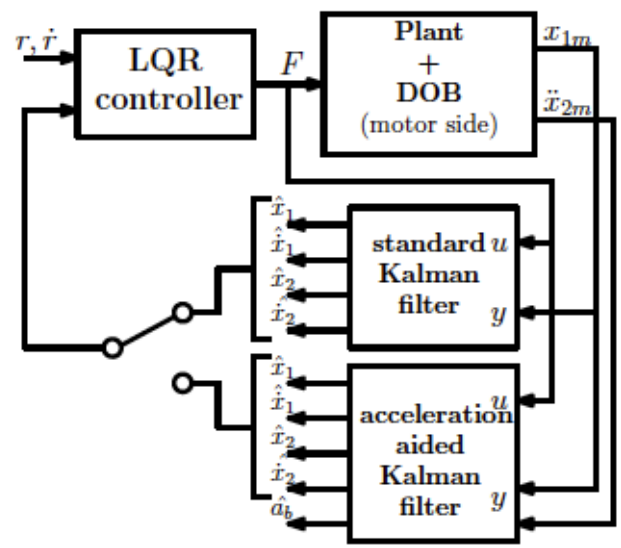
- Once the discrete–time models are available, it is easy to implement the prediction and correction steps, that are the same the KFs for both systems
  - It is worth noticing that the difference between the standard KF and the proposed aaKF is the availability of a second measurement form the plant (namely, the load acceleration) and the inclusion of a stochastic model of the noise and bias affecting such measurement.
  - aaKF makes the estimates more accurate and, in turn, the control more robust against possible mismatch between model and actual system, which is the typical critical aspect in standard LQG approach.
  - It is worth noticing that acceleration is a linear combination of motor and load position

LQR and LQG comparison

- Motor with DOB–compensated friction and force disturbances has been controlled with different state feedback controllers, all implemented in Matlab–Simulink, with a sample frequency of 1 kHz
  - LQR
  - LQG with standard, motor position–based KF
  - LQG with aaKF



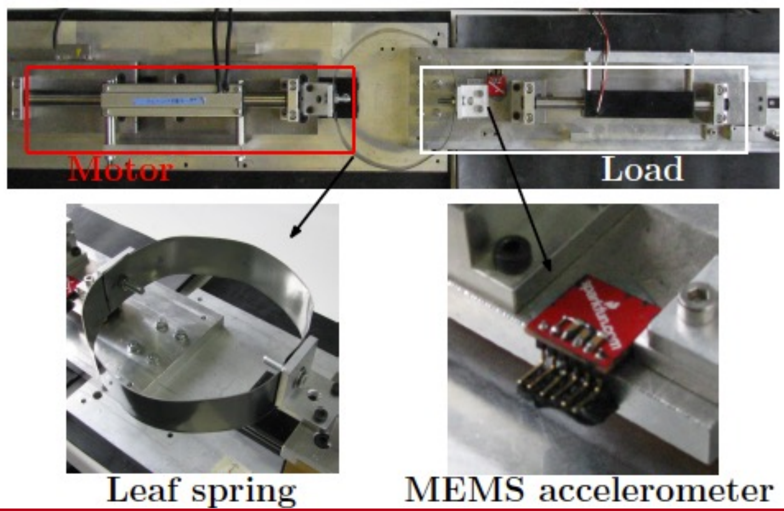
LQR control



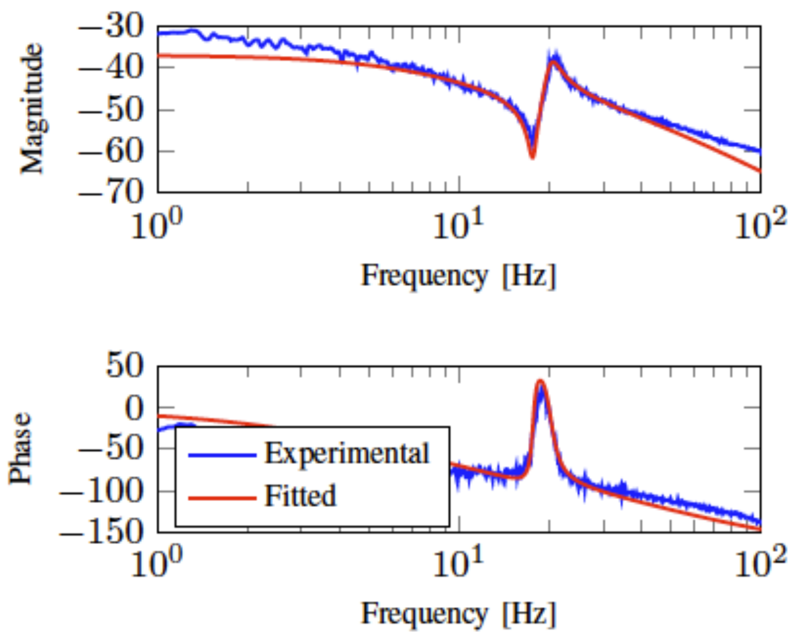
LQG controls with standard KF and aaKF

Experimental setup

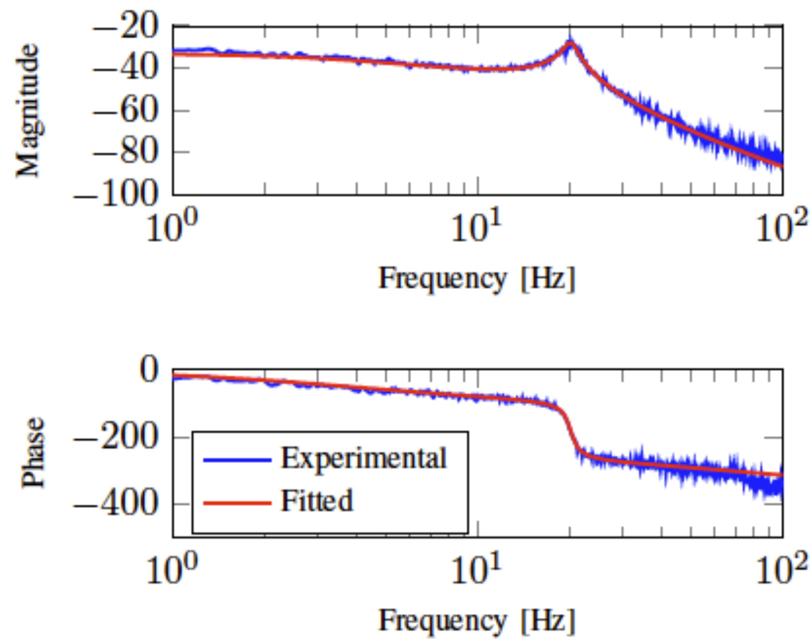
- The system is composed of a two linear motors, one used as actuator and the other as load, mounted on linear ball bearings and connected through a leaf spring
  - Both actuators have a position sensor
  - Load motor mounts a MEMS accelerometer (ADXL335)
- System parameters have been identified using a frequency–based approach



Experimental setup



Bode diagram of the plant with force as input and $\dot{x}_1$ as output

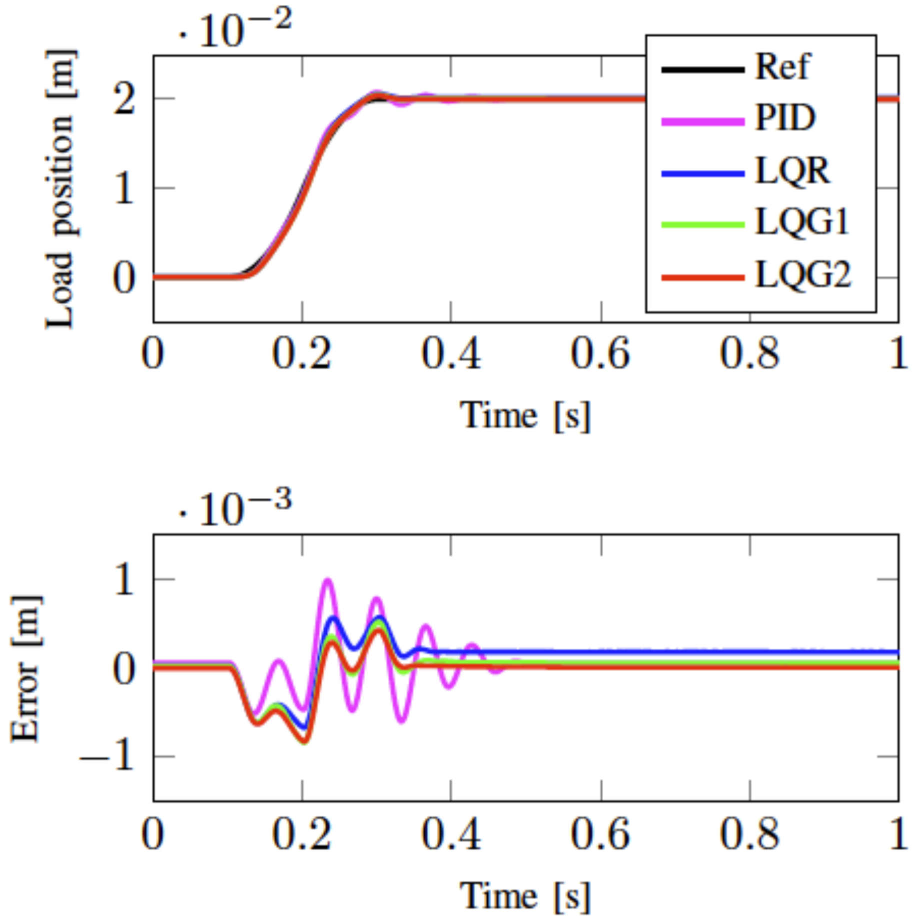


Bode diagram of the plant with force as input and $\dot{x}_2$ as output.

Experimental results

- In all experiments, the reference position is the same and it has an initial section with constant acceleration, followed by one at constant velocity and, finally by a constant deceleration.
- Both acceleration and deceleration are limited to a value that does not lead the actuator into saturation.
- LQG has been implemented in two ways: with standard KF (LQG1) and with aaKF (LQG2)
- In nominal condition (i.e. model perfectly matches the actual plant), LQR and LQGs outperform the PID control and they all exhibit similar performances

Experimental results

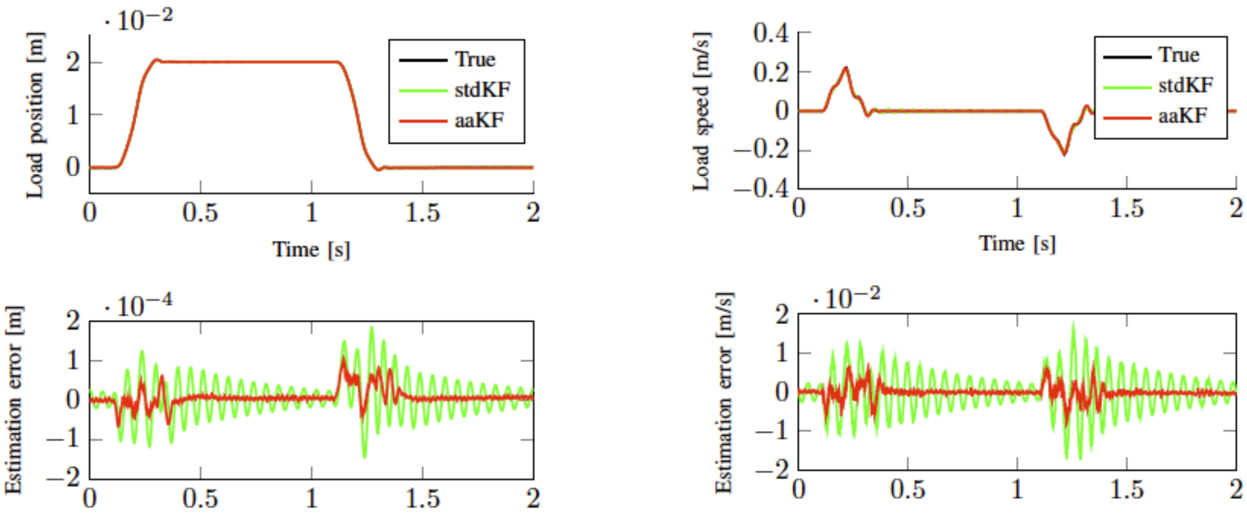


Control performance under nominal conditions

Experimental results

- Having a closer look to the estimation of the load side variables, it is possible to see that the aaKF-based LQG provides a much better estimate of position and velocity, compared to the standard LQG

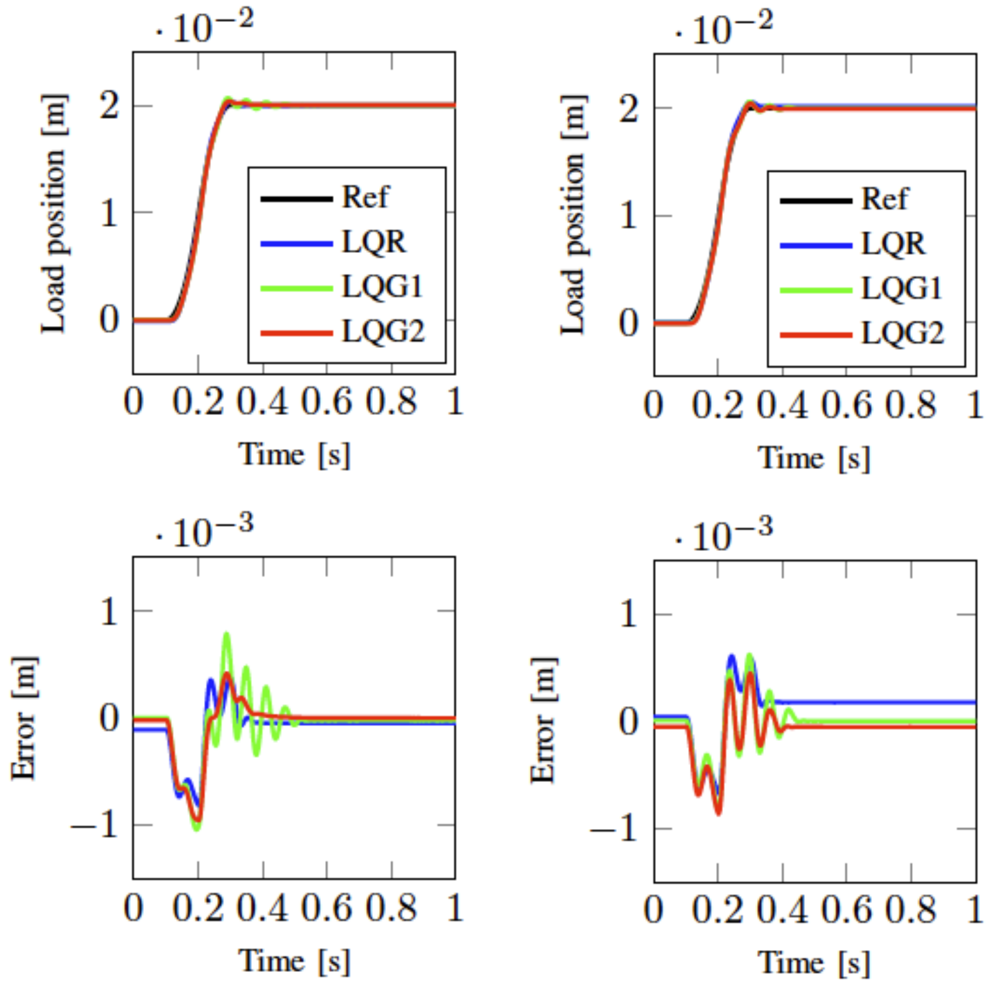
	RMS position error	RMS speed error
stdKF	5.1e-5 $[m]$	5.20e-3 $[m/s]$
aaKF	2.2e-5 $[m]$	1.60e-3 $[m/s]$



Experimental results
Robustness

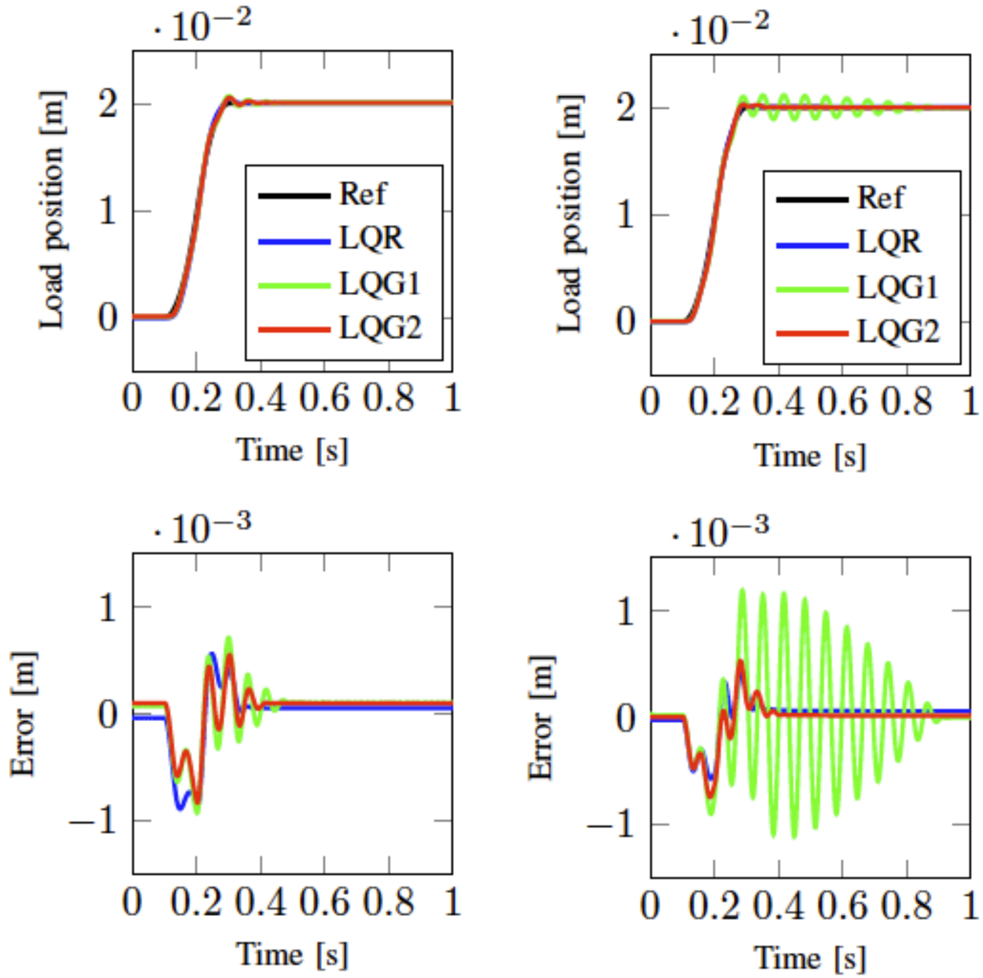
- Some differences are evidenced when the actual system parameters differ from those embedded into the models used for both feedback and estimator design.
- LQR is rather insensitive to large stiffness variations, while the control based on standard KF shows an oscillatory behaviour, which is much worse than that obtained with the proposed aaKF.
- The experimental results obtained by varying the load mass or the load-side friction are even more favorable to the aaKF-based LQG.

Experimental results
Robustness



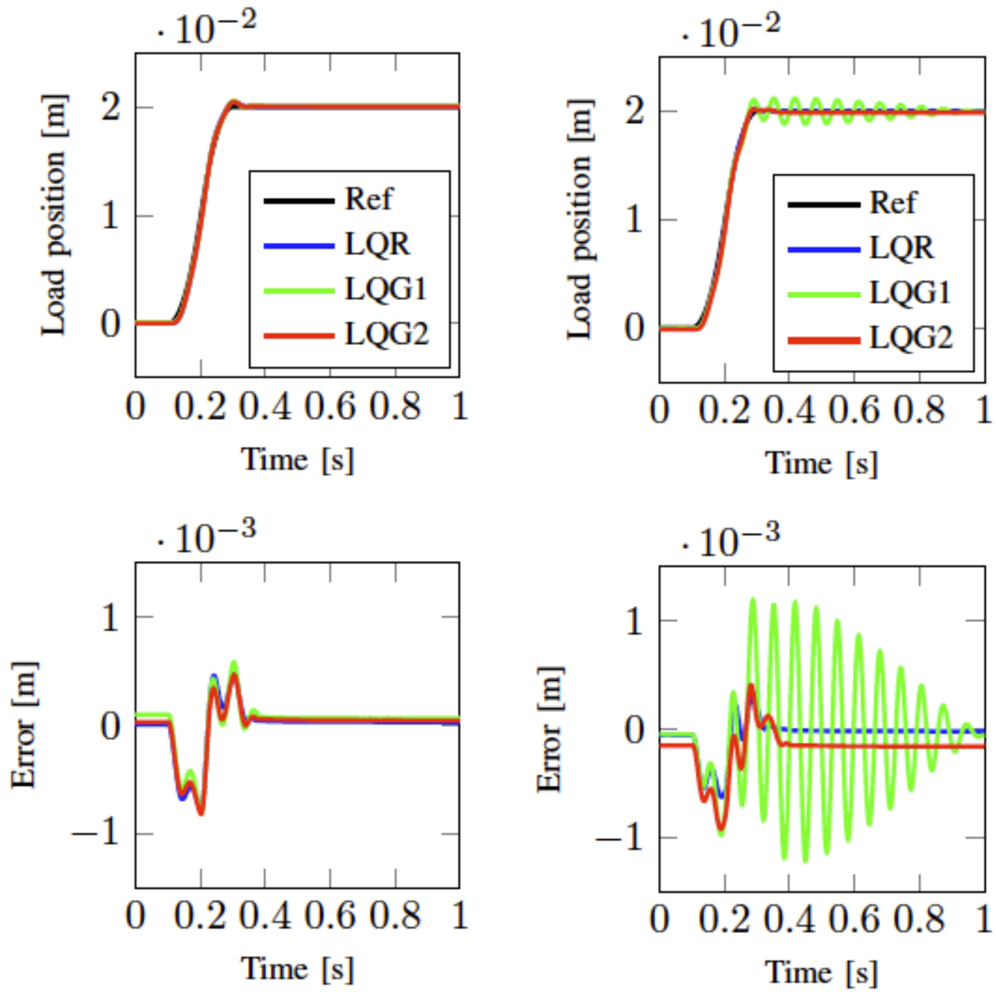
(a) $K/2$ (b) $2K$

Experimental results
Robustness



(a) $\frac{M_2}{2}$ (b) $2M_2$

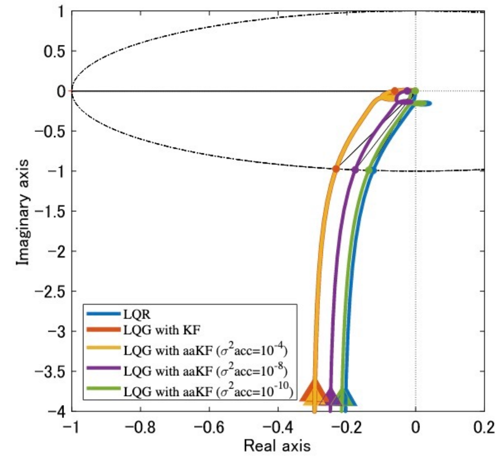
Experimental results
Robustness



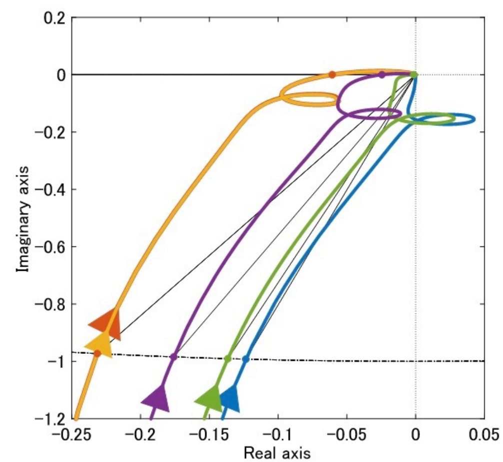
(a) $\frac{b_2}{2}$ (b) $2b_2$

Experimental results
Robustness

- The robustness of the proposed aaKF can be also analyzed by comparing the Nyquist plot of its loop transfer functions with those of the other controllers (LQR and LQG based on standard KF)
  - It can be seen that the LQG based on the aaKF matches the performance of a full-state feedback LQR, which is known to provide the best stability margins



(a) Nyquist diagram



(b) Enlarged view

Conclusions..

- We reported some results obtained by implementing a state feedback control of a two-mass-spring system, when the state estimate is obtained by using a KF which makes use of the load side acceleration.
- In addition to similar solutions found in literature, we explicitly accounted for the presence of bias and drift, always present when a low-cost MEMS accelerometer is used.
- As a result, the LQG control based on the proposed aaKF outperforms the standard one and, in terms of performance and robustness
- It achieves similar results of a standard LQR, based on the availability of the measurements of the full state.

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- R. Oboe and D. Pilastro, "Use of Load-Side MEMS Accelerometers in Servo Positioning of Two-Mass-Spring Systems," Proceedings of Industrial Electronics Society, IECON 2015 - 41st Annual Conference of the IEEE, November, 2015, pp. 4603–4608

Many thanks for your kind attention!