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Energy dissipation and hysteresis cycles in pre-sliding transients of kinetic friction

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The problem of transient hysteresis cycles due to pre-sliding friction is of importance for the analysis and identification of micro- and nano-positioning instruments and devices and their precisely controlled operation. The convergence to zero equilibrium under nonlinear frictional damping appears to bear nontrivial trajectory solutions, cf. [7], that can degrade the control performance by inducing multiple pre-sliding cycles before attaining final zero velocity. Obviously, even an unforced motion dynamics

$$m\frac{d^2x}{dt^2} + h\left[\frac{dx}{dt}, t\right] = 0, \quad \frac{dx}{dt}(0) \neq 0 \tag{1}$$

of the point mass m, which is damped by the nonlinear friction $h(\cdot)$ with hysteresis, requires a series of piecewise continuous solutions on the intervals $[t_i, \ldots, t_{i+1}]$, where i is the index of consecutive reversals of a relative displacement. The formal proof of stability and, therefore, type of convergence of $dx/dt \to 0$ in (1) is nontrivial and depends on the analytic form of frictional hysteresis map $h(\cdot)$ and its parametrization, also with respect to m. To find a suitable Lyapunov function candidate for (1) is not always possible for an assumed $h(\cdot)$. For the distributed Maxwell-slip friction model, which is equivalent to the Prandtl-Ishlinskii (PI) stop-type hysteresis operator and allows for describing the Coulomb friction with pre-sliding cf. [6], such Lyapunov-based analysis was demonstrated in [1]. However, for a finite elements realization, at least one PI element should have infinite stiffness, thus approaching $\text{sign}(dx/dt)$ for guaranteing convergence and avoiding stable limit cycles of (1). For more compact, in terms of dimension of the parameters space, and piecewise smooth friction models, like the well-celebrated Dahl one [3] and the hysteresis-area oriented tribological one [4], an easy-to-handle candidate for the Lyapunov function may not exist, or at least cannot be found with the given analysis tools. Motivated by the way of analysis [5] of mechanical systems with frictional hysteresis, we will consider the (x, h) phase-plane for which (1) can be transformed to

$$m \int\limits_{\dot{x}_i}^{\dot{x}_{i+1}} \dot{x} d\dot{x} = - \int\limits_{x_i}^{x_{i+1}} h(\dot{x}, t) dx = 0. \tag{2}$$

Assuming, without loss of generality, the pre-sliding initial condition $x_i(0) = 0$ and taking into account that $\dot{x}_i = \dot{x}_{i+1} = 0$ for each pair of the consecutive

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motion reversals, the evaluation of both integrals in (2) allows analyzing the pre-sliding hysteresis cycles. At each period of the pre-sliding cycles, the kinetic energy $E_k = 0.5\, m\dot{x}^2$ is balanced by the energy of hysteresis $E_h = \int_{x_i}^{x} h(\cdot)dx$, while the latter consists of the recuperative potential part E_p and dissipative damping part E_d, so that $E_h = E_p + E_d$. From the energy conservation viewpoint, it is obvious that the kinetic energy will grow, and thus $|\dot{x}(t)|$ increases as long as $\mathrm{sign}(\dot{x}) = -\mathrm{sign}(h)$, while $|\dot{x}(t)|$ starts to decrease as long as the sign of velocity and friction force becomes the same. Respectively, it applies $\max|\dot{x}| = |\dot{x}(t_0)|$, where $t_i < t_0 < t_{i+1}$ is the time instant of $h(\cdot) = 0$. The above developments allow calculating both, the energy dissipation during pre-sliding cycles driven by the hysteresis, and the size and period of these cycles. We will analyze two piecewise smooth pre-sliding hysteresis mappings, one of the Dahl model [3] and another one of the hysteresis-area based model, as it is provided in [2]. Both can be seen as a tribology- and engineering-motivated extension of the classical Coulomb friction law, that captures the pre-sliding hysteresis transitions and, this way, avoids discontinuity in $h(\cdot)$ at the velocity zero crossing. Outside the pre-sliding range and until the next t_i, both models saturate at the Coulomb friction level F_c and reduces the system dynamics (1) to the $m\ddot{x} = -F_c\, \mathrm{sign}(\dot{x})$ form.

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